

5D interpolation and regular upsampling: ill-suited or fit-for-purpose?

Mike J. Perz* and Peter W. Cary, Arcis Seismic Solutions, A TGS Company, Calgary, AB, Canada
mperz@arcis.com

Summary

5D minimum weighted norm interpolation (MWNI) is not suited for regular data upsampling according to a well-documented theoretical argument. However the reality is that 5DMWNI is often used successfully for this very task. We attempt to reconcile the theory to this paradoxical observation by showing that 5DMWNI may be used to successfully perform regular upsampling under certain restrictive conditions.

Introduction

5DMWNI has gained worldwide acceptance as a useful production processing tool, enjoying great popularity in Western Canada in particular. Recently various authors have discussed a certain algorithmic limitation in performing systematic upsampling across the cmp coordinates (Nagizadeh, 2010; Cary, 2011a; Wang et al., 2011). However others (Trad, 2008; Cary 2011b) have shown real data examples which indicate that the algorithm can produce excellent results for the same task, especially in the case of unstructured data acquired using a Megabin template. Unfortunately the aggregate message from these publications may have done more to confuse than to clarify from the perspective of the interpreter. The purpose of the present paper is to reconcile the apparent contradiction between theory and real-world observation, and thereby hopefully remove any lingering confusion about what the algorithm can and cannot do when it comes to regular upsampling of sparse data.

Theory and/or Method

The confusion arises because there are three competing factors which influence algorithm success for the regular upsampling task on any particular real data set. The primary factor pushing 5DMWNI toward failure is the fact that a certain algorithmic assumption is fundamentally inconsistent with the upsampling action, as discussed in detail below. On the other hand, there are two factors driving the algorithm towards success: first, simple wavenumber filtering can overcome this assumption incompatibility issue in the case of unstructured data, and second, irregularity in spatial sampling can diminish the severity of the same issue. All three factors are examined below.

(i) MWNI assumption inconsistency

The MWNI algorithm revolves around a certain inverse problem. In plain words, this inverse problem may be posed as: "Given the incomplete input data volume, find a complete data volume whose associated spatial Fourier spectrum is at once sparse and also fits the input data upon inverse Fourier transformation." Here the word "sparse" refers to a Fourier amplitude spectrum containing a relatively small number of high amplitude components. This sparsity constraint leads MWNI to reinforce any high amplitude Fourier components present in the incomplete input spectrum as it drives towards its final interpolated result. Unfortunately the upsampling task requires suppressing, rather than reinforcing, a certain high amplitude portion of the input spectrum associated with a phenomenon known as spectral replication (illustrated below), leading to unsatisfactory results. Figure 1 attempts to explain the problem via simple synthetic experiment. Figure 1a shows some simple 2D synthetic data consisting of multiple dipping events, some of which are spatially aliased. In order to perform 2:1 upsampling, these data are cast onto the finely sampled output grid (i.e., zero traces are interspersed between lines as shown in expanded view). Figure 1b shows the f-k spectrum associated with this fine output grid. The spectral

replicas are indicated by the dashed lines in Figure 1c, and are caused by the zero trace insertion. Proper upsampling entails zeroing out all of the dashed lines in Figure 1c and retaining only the solid lines associated with the primary spectrum. Unfortunately, the sparsity assumption seeks to preserve all of the dominant spectral elements, including those associated with the replicas, and therefore MWNI produces erroneous results (though not shown here, the interpolation generated very weak amplitude traces at the missing trace locations).

Although various algorithm modifications have been proposed to help overcome the problem, none of them are routinely used in the industry. Cary (2011b) shows that a certain frequency bootstrapping trick (which entails using the MWNI result from the lower, unaliased temporal frequencies to guide the algorithm at higher frequencies) does not completely solve the problem. Naghizadeh (2012) proposes an interesting technique to constrain the f-k spectrum of the final result so that it tends to create linear events radiating outward from the origin (i.e., from $f=k=0$). The technique gives good results on simple 2D and 3D synthetic data (Gao et al., 2012), but practical challenges would abound in implementing it in the full 5D case, and considerable testing would be required to assess its effectiveness on real data.

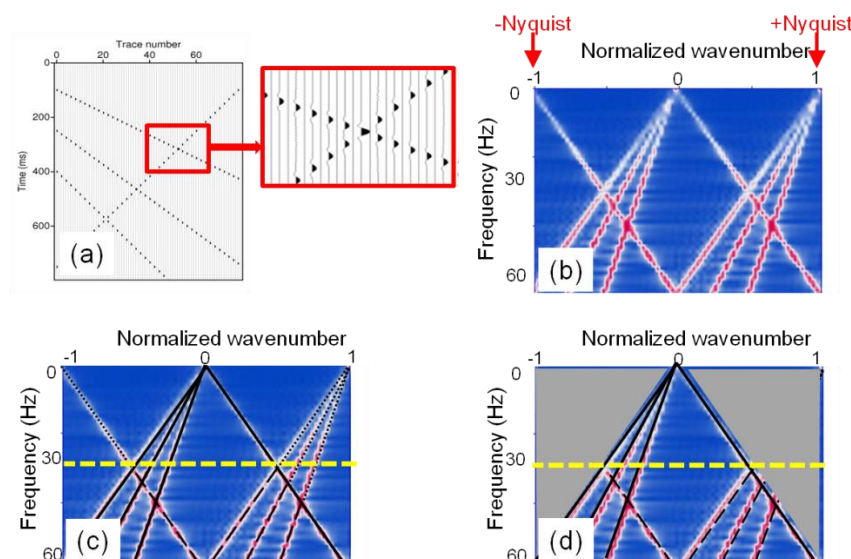


Figure 1: 2:1 upsampling experiment for 2D synthetic data. (a) Input data in x-t domain; (b) f-k spectrum associated with the finely sampled output grid shown in (a); (c) same as (b) except primary and replicated spectra are indicated by solid and dashed lines, respectively; (d) same as (c) except a wavenumber mask (grey polygons) has been applied.

(ii) Wavenumber masking for unstructured data

In the case of unaliased data, the situation depicted in Figure 1 is not so pessimistic. Spatial aliasing occurs when the spectral replicas encroach on the primary spectrum. In Figure 1c such aliasing is seen to occur for temporal frequencies above 30 Hz (i.e., below yellow dashed line). Note that the aliased portions of the replicas are indicated by long dashes while the unaliased parts are indicated by short dashes. For those temporal frequencies associated with the unaliased portion of the spectrum (i.e., 0-30 Hz), it is possible to explicitly zero out the energy at high wavenumbers in order to destroy the spectral replicas and therefore achieve good upsampling. Figure 1d shows an example of such an operation (called “wavenumber masking”), where the masked regions (grey polygons) are carefully chosen to preserve the primary spectrum. Note for frequencies above the 30 Hz cutoff, such simultaneous replica destruction and primary preservation is not possible, so the masking will only work for low frequencies on these steeply dipping synthetic data. On the other hand, the onset of aliasing occurs at temporal frequencies well beyond the maximum recorded frequency for most Western Canadian stratigraphic data sets. In such cases masking is possible across the entire passband, allowing 5DMWNI to give good results as we will show later on.

(iii) Effect of random spatial sampling

The preceding tests studied regularly sampled data in one spatial dimension. For land acquisition in four spatial dimensions, surveys often exhibit sufficient irregularity in sampling to significantly diminish the strength of the spectral replicas. This in turn can lead to improved results for upsampling compared to the case where the sampling is regular. Figure 2a shows a perfectly regular 3D control geometry inspired by the real crooked land geometry displayed in Figure 2b (the associated seismic data set from the Canadian foothills was studied by Wang et al. (2011)). As shown in these two figures, the shot interval (312 m) is four times coarser than the receiver interval, resulting in a highly elongated natural CMP bin (39 x 156 m) and the interpolation task entails 4:1 upsampling onto the final square 39 x 39 m CMP grid. The fold plot associated with the regular control geometry on this final grid is shown in Figure 2c. Note that three out of four inlines are missing (though not shown here, the fold plot for the real geometry shows much more random scatter of midpoints). Figure 2d is a geological cartoon showing the four steeply dipping events that were used in the synthetic modeling experiment. Note that the two events depicted in green are spatially aliased along the coarsely sampled crossline direction (indicated by heavy black dashed line) given the frequency content of the data. Forward modeling was performed to generate prestack synthetic data volumes for both the irregular and regular geometries. These data volumes were input to 5DMWNI and the results for a single common offset vector (COV) are analyzed along crossline 300 (heavy dashed lines in Figures 2c and 2d). Azimuth of the COV relative to the crossline direction is shown by mock source-receiver pairs in Figure 2d. Figures 2e and 2f show the images after 5DMWNI for the regular and irregular geometries, respectively (practical difficulties precluded us from sampling the same lateral portion of the earth model in both experiments but this does not alter our result interpretation). The left-to-right dipping aliased event in the regular sampling case of Figure 2e shows very strong artifacts, while the image is somewhat improved, but still erroneous, in the irregular geometry case of Figure 2f. This synthetic experiment confirms that randomness in 4D sampling associated with the irregular acquisition helps to at least partially mitigate the upsampling problem.

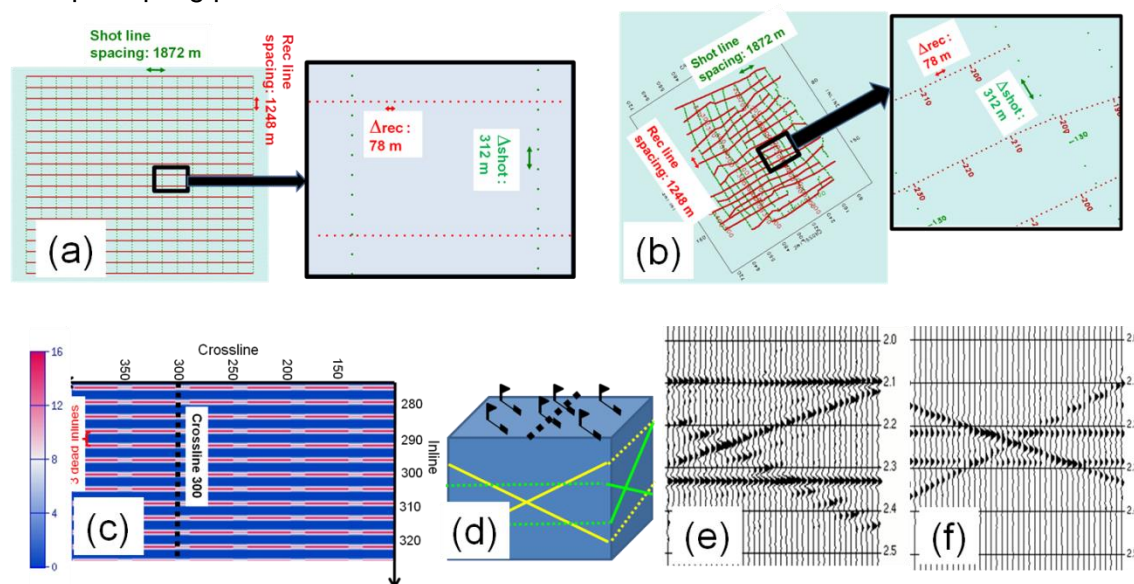


Figure 2: Synthetic experiment illustrating the effect of irregular sampling on 5DMWNI success. (a) surface plot of regular control geometry; (b) real irregular geometry whose average source/receiver line intervals and spacings were used to seed the control geometry in (a); (c) subsurface fold plot for regular geometry in (a); (d) cartoon showing earth model for synthetic data; (e) 5DMWNI output corresponding to control geometry in (a); (f) 5DMWNI output corresponding to real data geometry in (b).

Examples

The examples below illustrate the push-pull of the three factors discussed above. First we examine the heavily structured and spatially aliased data set associated with the real survey geometry from Figure 2. Figure 3a shows the result of performing the regular 4:1 upsampling using 5DMWNI. The existence of strong striping artifacts reveal 5DMWNI's breakdown, despite the mitigating effect associated with

the irregular sampling (the along-crossline direction shown in Fig. 2d runs straight up-and-down in this present figure). Figure 3b shows the corresponding result obtained using a coherence-guided approach (Wang et al., 2011) which gives a much improved image. Based on the discussion above, we would expect further degradation in the 5DMWNI image if the acquisition was more regular.

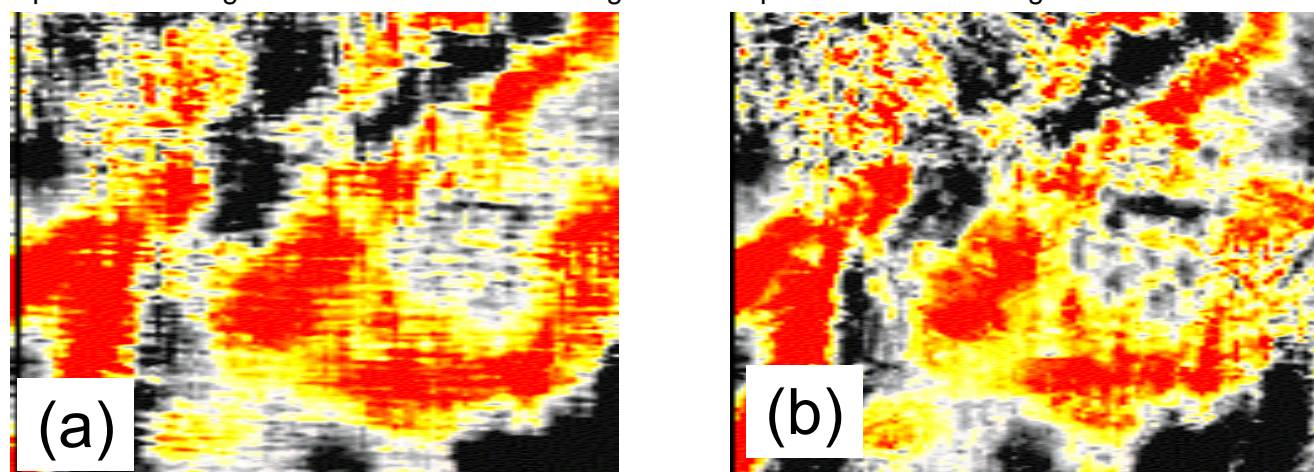


Figure 3: Timeslices of structure stack from the heavily structured real data set associated with the crooked survey studied in Fig. 2. (a) result after 5DMWNI; (b) result after coherence-guided interpolation.

In the second example we show simple, unaliased data from Western Canada, again under a regular 4:1 upsampling scenario. The timeslice in Figure 4a shows the result from a 5DMWNI run which employed a modest amount of wavenumber masking, Figure 4b shows the corresponding result after applying more aggressive masking. In this case the aggressive masking has eliminated most of the striping artifacts visible in Figure 4a. Thus it is clear that masking can help produce good results in the case of upsampling unstructured data.

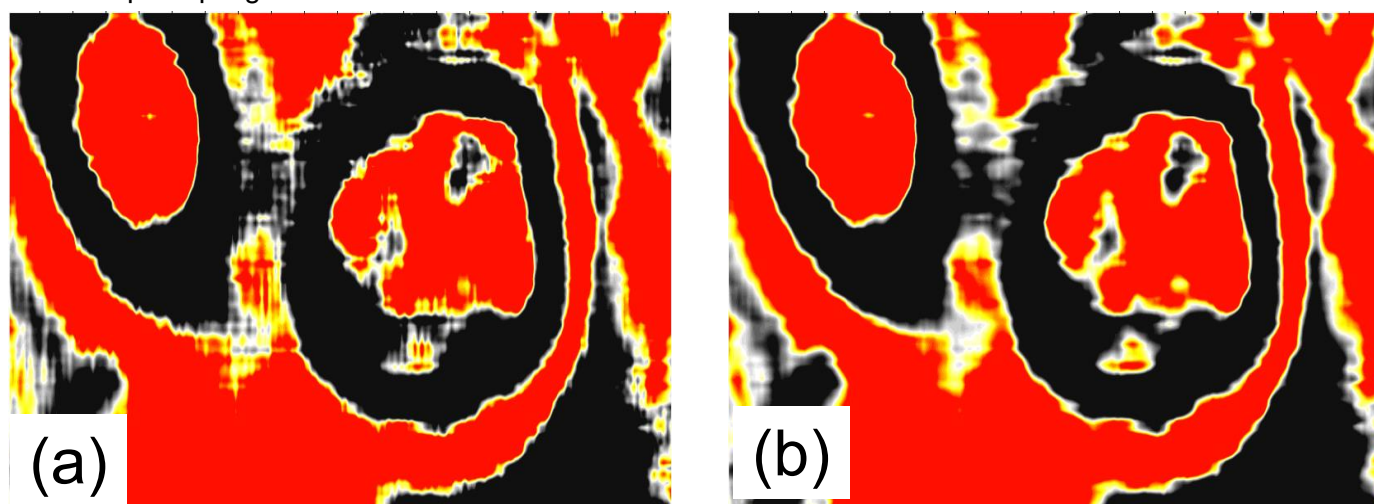


Figure 4: Timeslices of structure stack from an unstructured data set. (a) result after 5DMWNI using small amount of wavenumber masking; (b) result after more aggressive masking.

Conclusions

Although the 5DMWNI algorithm's assumption of wavenumber sparsity is fundamentally inconsistent with the regular upsampling task, 5DMWNI may still give good results in cases where the data are unstructured and/or the sampling is sufficiently randomized.

Acknowledgements

We thank Husky Energy Inc. and an anonymous data donor for granting us showrights for the real data examples.

References

- Cary, P.W., 2011a, An investigation into interpolation with the MWNI algorithm: CSEG April Lunchbox Geophysics presentation.
- Cary, P.W., 2011b, Aliasing and 5D interpolation with the MWNI algorithm: 81st Annual International Meeting, SEG, Expanded Abstracts, 3080-3083.
- Gao, J., A. Stanton, M. Naghizadeh, M. Sacchi, and X. Chen, 2012, Convergence improvement and noise attenuation considerations for beyond alias projection onto convex sets reconstruction: *Geophysical Prospecting*, doi: 10.1111/j.1365-2478.2012.01103.x
- Naghizadeh, M., 2012, Seismic data interpolation and denoising in the frequency-wavenumber domain: *Geophysics*, 77, no. 2, V71-V80.
- Naghizadeh, M., and M. D. Sacchi, 2010, On sampling functions and Fourier reconstruction methods: *Geophysics*, 75, no. 6, WB137–WB151.
- Trad, D., M. Hall., and M. Cotra, 2008, Merging surveys with multidimensional interpolation: CSEG National Convention, Convention Abstracts, 172-176.
- Wang, J., D. Quinn, D. Negut, D. Ganley, M. Perz, M. Kola-Ojo, A. Truong, and D. Emery, 2011, The effect of input data sampling on prestack interpolation efficacy: lessons learned from a sparsely shot and heavily structured 3D data set: 81st Annual International Meeting, SEG, Expanded Abstracts, 3075-3078.