

# Kirchhoff imaging in a matrix form. Part I: modelling

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## Summary

Kirchhoff prestack modelling may be computed using a matrix-vector multiplication. This paper presents a visual method of the constructing the modelling operator,  $G$ , in an explicit, matrix form, that contains diffractions. Once the matrix is created, modelling can be repeated by multiplication of the matrix to the reflectivity models.

## Introduction

Each subsurface point may be considered as a scatterpoint that creates diffracted energies. Kirchhoff migration collapses all diffracted energies back to the scatterpoints. Kirchhoff prestack modelling, on the other hand, spreads the energy from the scatterpoint to the seismic data. Kirchhoff prestack time modelling uses the Double Squares Root (DSR) equation to calculate the modelling time for each sample in each trace. Proper weighting functions, filtering, interpolation, dip, and aliasing controls are also necessary considerations in Kirchhoff modelling.

Let's the earth reflectivity model be defined by  $m$ , and the Kirchhoff modelling operator by  $G$ . The modeled seismic data,  $d$ , is given by,

$$d = Gm. \quad (1)$$

Construction of prestack Kirchhoff modelling matrix,  $G$ , is shown. For detailed information about this method readers are being referred to Claerbout (2005) and Yousefzadeh and Bancroft (2010, 2011).

## Constructing the $G$ matrix

In order to create synthetic seismic data with the Kirchhoff modelling operator (equation 1), a form of  $G$ , in either an implicit or an explicit form is required. When in the implicit form  $G$  is a computer code, in the explicit form  $G$  is a matrix. The matrix is multiplied directly to the reflectivity model,  $m$ , to produce data,  $d$ . The modelling matrix may be saved in the computer memory and multiplied with the model several times when solving least squares Kirchhoff migration by the conjugate gradient method.

The construction of  $G$  matrix assumes a 2D earth model with scatterpoints throughout the subsurface. Our example will consider only one scatterpoint located at the surface. Let's consider a seismic source right on the top of this scatterpoint and eight receivers on both sides of the source, as shown in Figure 1.



Figure: 1. A 2D model including one source (red triangle) on the top of a scatterpoint (blue circle) and eight receivers (flags) on the earth surface.

This model can be presented by matrix  $m$ :

$$m = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2)$$

The scatterpoint with the amplitude of 1 is located at row #1 and column # 5. For simplicity we show this model by a 1 by 1 matrix as  $\tilde{m}_{1,5} = [1]$ . Assuming constant velocity background and ignoring any amplitude change during wave propagation, the diffraction matrix for a scatterpoint at the row #1 and column # 5 of the model space,  $\tilde{G}_{1,5}$ , will have a diffraction hyperbola in eight traces and six samples per trace with an assumed sampling rate:

$$\tilde{G}_{1,5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

The data created with this geometry will produce a similar matrix as illustrated in equation 4:

$$d_{1,5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (4)$$

The modelling operator for a scatterpoint in row #1 and column # 5 of the model space,  $\tilde{G}_{1,5}$ , has the same shape as the data,

$$\tilde{G}_{1,5} = d_{1,5}. \quad (5)$$

The  $G$  matrix that created  $d$  is a multidimensional matrix that requires one diffraction for each scatterpoint. Multiplication of the modelling operator with the model should be mathematically possible and creates the data,

$$\tilde{G}_{1,5} \tilde{m}_{1,5} = d_{1,5}. \quad (6)$$

However,  $\tilde{G}_{1,5}$  and  $d_{1,5}$  are matrices with  $N_t$  by  $M_{trc}$  size, where  $N_t$  is the number of samples per trace and  $M_{trc}$  is the number of traces, and  $\tilde{m}_{1,5}$  is a 1 by 1 matrix. Multiplication in equation 6, becomes possible by vectorizing the  $\tilde{G}_{1,5}$  and  $d_{1,5}$  matrices. In order to have  $\tilde{G}_{1,5}$  (or  $d_{1,5}$ ) as a vector,  $\tilde{G}_{v1,5}$  (or  $d_{v1,5}$ ), we put column of  $i + 1$  of  $\tilde{G}_{1,5}$  (or  $d_{1,5}$ ) matrix below column  $i$  of that matrix for all columns,

$$\tilde{G}_{v1,5} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad (7), \quad \text{and} \quad d_{v1,5} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ 0 \end{bmatrix}. \quad (8)$$

Now  $\tilde{\mathbf{G}}_{v1,5}$  and  $\mathbf{d}_{v1,5}$  are vectors with  $N_t \times M_{trc}$  elements and we can rewrite equation 6 as the following matrix-vector multiplication:

$$\tilde{\mathbf{G}}_{v1,5} \tilde{\mathbf{m}}_{1,5} = \mathbf{d}_{v1,5}. \quad (9)$$

Now suppose that our model, which has  $N_x$  elements in each row, has other scatterpoints in the first row. To accomplish operator-matrix multiplication as we did for the case of one scatterpoint we should be able to do the following multiplication:

$$\tilde{\mathbf{G}}_{1,:} \tilde{\mathbf{m}}_{v1,:} = \mathbf{d}_{v1,:}, \quad (10)$$

where  $\tilde{\mathbf{G}}_{1,:}$  is the corresponding modelling operator for all scatterpoints in the first row of the model,  $\tilde{\mathbf{m}}_{v1,:}$ , and  $\mathbf{d}_{v1,:}$  is the resulted data. This is possible by defining  $\tilde{\mathbf{G}}_{1,:}$  a  $N_t \times M_{trc}$  by  $N_x$  matrix by:

$$\tilde{\mathbf{G}}_{1,:} = [\tilde{\mathbf{G}}_{v1,1} \tilde{\mathbf{G}}_{v1,2} \dots \tilde{\mathbf{G}}_{v1,5} \dots \tilde{\mathbf{G}}_{v1,N_x}]. \quad (11)$$

Then multiplication of  $\tilde{\mathbf{G}}_{1,:}$  by  $\tilde{\mathbf{m}}_{v1,:}$  gives the resulted data from modelling of all points in the first row of the reflection matrix in equation 10.

This is the result of the seismic experiment for the first row of model. Result of modelling of other rows in the image domain must be added to the current data as well. The mentioned procedure can be extended for the other rows of model by defining the modelling matrix for all elements in each row, and placing all next to each other,

$$\mathbf{G} = [\tilde{\mathbf{G}}_{v1,1} \tilde{\mathbf{G}}_{v1,2} \dots \tilde{\mathbf{G}}_{v1,N_x} \tilde{\mathbf{G}}_{v2,1} \tilde{\mathbf{G}}_{v2,2} \dots \tilde{\mathbf{G}}_{vN_z,N_x}], \quad (12)$$

where  $N_x$  and  $N_z$  are the numbers of columns and rows in the model, respectively.

Model needs to be vectorized by the same method,

$$\mathbf{m}_v = \begin{bmatrix} \tilde{\mathbf{m}}_{v1,:} \\ \tilde{\mathbf{m}}_{v2,:} \\ \vdots \\ \tilde{\mathbf{m}}_{vN_z,:} \end{bmatrix}. \quad (13)$$

Each column of the  $\mathbf{G}$  represents a point in the migrated image. Therefore, matrix  $\mathbf{G}$  includes  $N_t \times M_{trc}$  rows and  $N_x \times N_z$  columns. The result of multiplication of  $\mathbf{G}$ , a  $N_t \times M_{trc}$  by  $N_x \times N_z$  matrix, to the  $\mathbf{m}_v$ , a  $N_x \times N_z$  vector is  $\mathbf{d}_v$ , a  $N_t \times M_{trc}$  vector:

$$\mathbf{G} \mathbf{m}_v = \mathbf{d}_v \Rightarrow \begin{matrix} N_t \\ \times \\ M_{trc} \end{matrix} \left\{ \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}}_{N_x \times N_z} \right\} \begin{matrix} N_x \\ \times \\ N_z \end{matrix} \left\{ \begin{bmatrix} \dots \\ \dots \\ \dots \\ \dots \end{bmatrix} \right\} = \begin{matrix} N_t \\ \times \\ M_{trc} \end{matrix} \left\{ \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \right\}. \quad (14)$$

The resulted  $\mathbf{d}_v$  vector must be reshaped to show the data in the conventional form. Calculation of  $\mathbf{G}$  must include modelling weight (instead of having just ones) and other parameters. Weight is added to  $\mathbf{G}$  matrix by replacement of ones with proper amplitude weights. By using a ray tracing program instead of DSR equation, this technique can be extended to the Kirchhoff prestack depth modelling.

## Example and feasibility of using $G$ matrix

When  $G$  is available in the matrix form, it can be used in the least squares prestack Kirchhoff migration. It seems easier to use the matrix form when we need to perform Kirchhoff migration and modelling repeatedly. It is also feasible to add some regularization to inversion.

$G$  is a very large matrix. The number of elements in  $G$  is equal to the number of elements in the model multiplied by the number of elements in data. Suppose the assumed geometry includes only one source, 15 receivers, 15 CMPs, 200 pseudo depth/time samples, and the data has 300 samples per trace.  $G$  will be a 3000 by 4500 matrix. This matrix is shown in Figure 2.

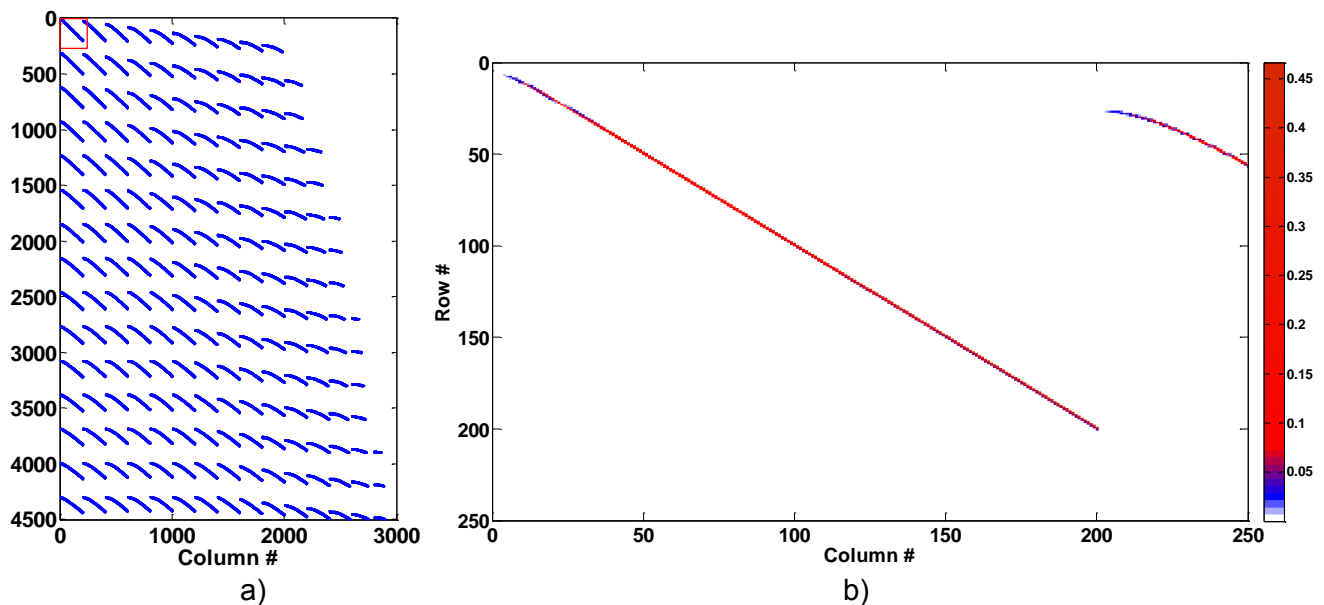


Figure 2: a) Non-zero elements of matrix  $G$ , b) Close up of the first 250 rows and columns.

## Conclusions

A technique for creating Kirchhoff modelling matrix without the requirement of going into the mathematics of Kirchhoff modelling or Green functions is introduced. Once it is calculated, modelling can be repeated just by multiplication of the matrix to the model vector. However, the modelling matrix is very large which make it impractical for many real datasets.

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