

Rank-Reduction-Based Trace Interpolation

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Summary

In previous papers we described a family of multidimensional filters to suppress random noise based on matrix-rank reduction of constant-frequency slices. Here we extend these filters to perform multidimensional trace interpolation. This requires rank reduction when some, perhaps most, of the matrix elements are unknown, a procedure called *matrix completion* or *matrix imputation*. We show how this new interpolator improves the spatial resolution of 3D data when applied prior to prestack migration.

Introduction

For optimal 3D prestack migration, input traces should be evenly and densely sampled in:

- Inline midpoint
- Crossline midpoint
- Offset
- Azimuth

Otherwise one gets poor cancelation of overlapping migration impulse responses, resulting in migration artifacts (Gardner and Canning, 1994). Other benefits of an even and dense trace sampling are that it:

- Reduces the acquisition footprint
- Reduces spatial aliasing
- Improves multiple removal
- Improves the continuity of shallow events
- Improves amplitude variation with offset (AVO) analysis

Due to physical and financial constraints, acquisition geometries rarely deliver a perfect spatial sampling. Thus in recent years we've seen a surge in interest in prestack trace interpolation to improve spatial sampling prior to migration. Some popular interpolation methods for this are:

MWNI Minimum Weighted Norm Interpolation (Liu, 2004; Liu and Sacchi, 2004; Trad 2009)
POCS Projection Onto Convex Sets (Abma and Kabir, 2006)
ALFT Anti-Leakage Fourier Transform (Xu and Phan, 2004; Xu, Zhang, et al, 2004)

Abma (2009) gives a brief comparison of these methods. All of them interpolate simultaneously in multiple spatial dimensions, a far more powerful approach than cascading a one-spatial-dimension interpolator. Here we present a new multidimensional interpolation algorithm based on matrix-rank reduction working on constant-frequency slices, and demonstrate its use on a 3D data set.

Method

We use a strategy called *matrix-rank reduction* (see, for example, Trickett, 2003), also called truncated singular-value decomposition, principal-component analysis, subspace filtering, and many other names. The singular-value decomposition allows one to decompose a p -by- p matrix \mathbf{A} into the sum of p matrices of rank one, called *weighted eigenimages*:

$$\mathbf{A} = \mathbf{I}_1 + \mathbf{I}_2 + \dots + \mathbf{I}_p, \quad \|\mathbf{I}_i\|_2 \geq \|\mathbf{I}_{i+1}\|_2$$

A rank- k approximation to matrix \mathbf{A} is found by summing the first $k < p$ weighted eigenimages:

$$F_k(\mathbf{A}) = \mathbf{I}_1 + \mathbf{I}_2 + \dots + \mathbf{I}_k$$

Rank reduction is useful for noise suppression as coherent energy tends to fall into the first few eigenimages, while random energy is more evenly distributed across all eigenimages. Here's how it can be used to suppress random noise on constant-frequency slices of a multidimensional grid of traces (Trickett & Burroughs, 2009):

Take the Discrete Fourier Transform (DFT) of each trace in the grid.
 For each frequency within the signal band...

- {
- 1. Place the complex trace values for this frequency into a matrix \mathbf{A} (somehow).
- 2. Reduce the matrix to rank k .
- 3. Recover each trace value from the rank-reduced matrix by averaging all elements where that value was originally placed in matrix \mathbf{A} .
- }

Take the inverse DFT of each trace.

The method is complete once we know how to form matrix \mathbf{A} . Given a one-dimensional series of n traces having a constant-frequency slice $c_i, i = 1 \dots n$, we form a complex-valued Hankel matrix

$$\mathbf{A} = \begin{bmatrix} c_1 & c_2 & \dots & c_{n-m+1} \\ c_2 & c_3 & \dots & c_{n-m+2} \\ \vdots & \vdots & \ddots & \vdots \\ c_m & c_{m+1} & \dots & c_n \end{bmatrix}, \quad m \approx \frac{n}{2}$$

The resulting noise suppression is called f-x Cadzow filtering (Trickett, 2008). Two-dimensional Cadzow filtering forms a Hankel matrix of Hankel matrices. Three-dimensional Cadzow filtering forms a Hankel matrix of Hankel matrices of Hankel matrices, and so on for any number of spatial dimensions.

This is not the only way to form matrix \mathbf{A} . Trickett and Burroughs (2009) describe two other strategies, referred to as eigenimage and hybrid eigenimage-Cadzow.

How can we modify these noise-suppression filters to perform trace interpolation? The central problem to be solved is this:

Perform matrix rank reduction when some, perhaps most, of the matrix elements are unknown.

This is called *matrix completion* or *matrix imputation*, and many algorithms have recently been developed to solve it (Chen and Suter, 2004; Kurucz et al, 2007; Olson and Oskarsson, 2009).

3D Example

For prestack 3D data we must decide which domain to filter in. Trad (2009) lists various options. Here we interpolate in the inline and crossline midpoint, offset, and azimuth dimensions. Commercially this is often called “5D interpolation” (frequency plus four spatial dimensions), although mathematically the interpolation is carried out only in the four spatial dimensions. The natural rank-reduction filter for this is f-wxyz Cadzow, also referred to as C^4 .

Another possible 5D interpolation domain is common-offset vector (Cary, 1999), where the spatial dimensions are inline and crossline midpoint, and inline and crossline offset.

Figures 1 and 2 show the results of prestack interpolation of a 3D volume. Although the noise-suppressing power of these filters is valuable for cleaning up the data, it's the interpolation of near-offset traces that produces a dramatic reduction in the acquisition footprint, and consequently a significant improvement in spatial resolution.

Conclusions

Multidimensional rank-reduction-based random-noise suppression can be extended to perform trace interpolation. This new interpolator can be used to improve the spatial sampling of prestack seismic data, thus improving the resolution of the final seismic section. It seems particularly effective at suppressing the acquisition footprint.

Abma (2009) compared the properties and performance of three prestack interpolation methods. It would be interesting to see how this new method matches up.

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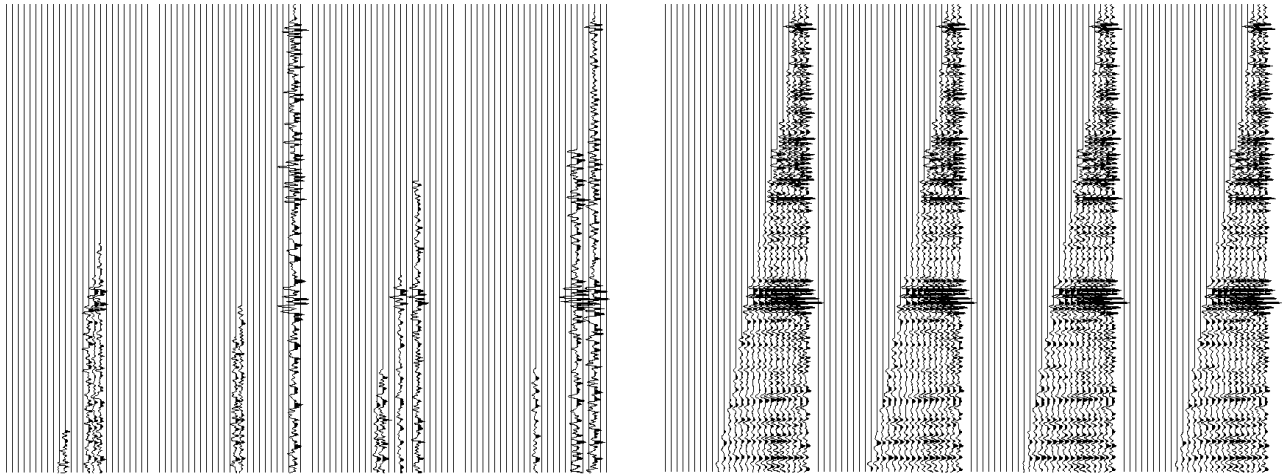


Figure 1: A 3D common-midpoint gather separated into four azimuth sectors, (left) before and (right) after interpolation. Data courtesy of Conoco-Phillips.

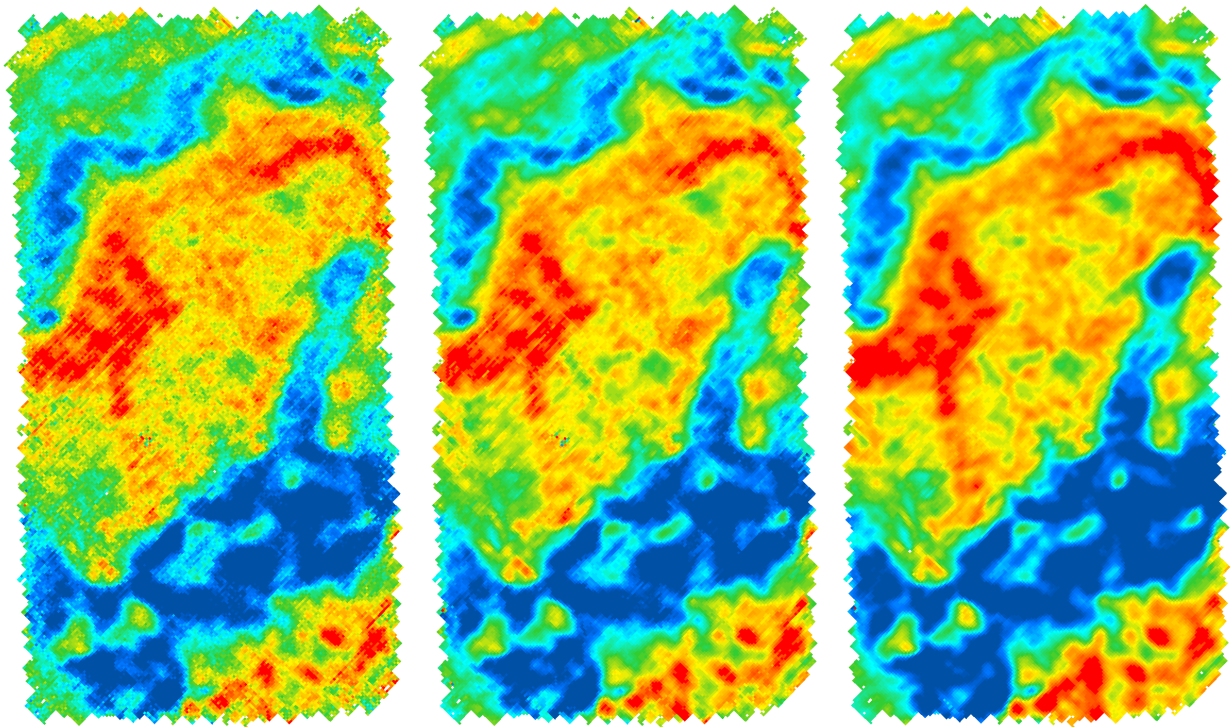


Figure 2: Left is a time slice of a 3D structure stack. Note the random noise and diagonal acquisition footprint. Middle has prestack rank-reduction-based noise suppression but no interpolation. Noise is reduced but the footprint remains. Right has prestack interpolation. The footprint is now mostly removed, resulting in improved spatial resolution. Data courtesy of Conoco-Phillips.