Are Geostatistical Gaussian Realizations Equi-Probable?

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Summary

Realizations of geostatistical Gaussian simulations are routinely used for the characterization and uncertainty assessment of reservoir properties such as porosity and permeability. Gaussian simulation methods are popular because they are relatively easy to implement and they can produce reasonably realistic results. Another important motivation for using geostatistical Gaussian realizations is that they are said to be generated with equal probability. Equal probability of realizations is important because it is required for unbiased uncertainty assessment and proper uncertainty propagation through transfer functions such as flow simulators. However, geostatistical Gaussian realizations are at the center of a dilemma.

On the one hand, any geostatistician can testify of the equi-probability of geostatistical realizations by observing that all realizations (as generated by the same Gaussian method using same grid, same data, and same parameter settings) look statistically very similar one to another. They all display similar patterns and similar spatial variability. Indeed, they all have similar reproduction of spatial statistics such as the variogram.

On the other hand, the Gaussian simulation methods (e.g. LU and Sequential) used to create geostatistical realizations are mathematically designed to generate realizations from a multivariate Gaussian distribution. Realizations from multivariate Gaussian distributions are not equi-probable, but instead follow a probability density characterized by the so-called bell shaped multi-dimensional surface. Realizations similar or close to the mean vector of the multivariate distribution are more probable and should be more frequently observed than realizations very dissimilar, or distant to the mean vector. In practice, this is never observed with geostatistical realizations. In fact, only realizations distant to the mean vector are observed. This is known as the typical non-smoothness of geostatistical realizations.

The above paradox is source of confusion among practitioners of geostatistics. This paper revisits the concept of equi-probability associated with the generation of geostatistical Gaussian realizations. The theory of multivariate Gaussian probability distribution will be briefly reviewed with emphasis on the Mahalanobis metric as a tool for measuring the distance between a realization and the mean vector. Numerical examples will be used to illustrate that Gaussian realizations, as generated by LU simulation method, are not equi-probable. It will be also shown that the probability of each realization quickly becomes very small as the number of dimensions (number of grid blocks) in the realization vector increases. The "geostats paradox" will be solved by showing that using random numbers for sampling multivariate Gaussian distribution of large vector size (many grid blocks) results in realizations that are preferentially located very far from the mean vector (with large Mahalanobis distance). These far out regions of the multivariate Gaussian distribution are characterized by a gentle sloping of the density surface where one can practically assume equi-probabilities. The relationship between probability of a realization and its quality of variogram reproduction will also be examined. It will be shown that the variogram reproduction for realizations in those regions is more characterized by random fluctuations as oppose to more important variations related to differences in the probability of the realizations which happens when realizations are closer to the mean vector.

Figures, equations, tables, discussions, conclusions, and references will complete the paper.