

# **Time-varying Wavelet Estimation and Deconvolution**

Mirko van der Baan\*

University of Alberta, Dept. of Physics, Edmonton, Canada

Mirko.VanderBaan@ualberta.ca

## **Summary**

Phase mismatches sometimes occur between final processed sections and zero-phase synthetics based on well logs, despite best efforts for controlled-phase acquisition and processing. Kurtosis maximization by constant phase rotation is a statistical method that can reveal the phase of a seismic wavelet. It is sufficiently robust that it can even detect time-varying phase changes. Phase-only corrections can then be applied by means of a time-varying phase rotation. Alternatively amplitude and phase deconvolution can be achieved using time-varying Wiener filtering. Time-varying wavelet extraction and deconvolution can also be employed as a data-driven alternative to amplitude-only inverse-Q deconvolution.

## **Introduction**

Controlled-phase acquisition and processing plays an important role in current acquisition and processing strategies (Trantham, 1994). Deterministic corrections such as for debubbling and for attenuation-related dispersion are favored over statistical approaches. Nonetheless, despite best efforts to control the phase of the wavelet during the entire acquisition and processing sequence, phase mismatches regularly occur between final processed data based on deterministic zero-phase shaping and zero-phase synthetics created from well logs. The existing well logs are often used in these cases as ground truth, and a further phase correction is applied to the data such that they match the zero-phase synthetics.

Unfortunately, well logs are not always available and different wells can predict different phase corrections. It is also possible that the phase mismatch varies with time. There is therefore a need for a statistical approach to estimate the phase of the wavelet from the data alone, yielding complementary information, and to serve as an additional quality control.

Levy and Oldenburg (1987), Longbottom et al. (1988) and White (1988) describe such a technique for stationary data. Their method is based on a simplification of the blind deconvolution method proposed by Wiggins (1978). They search for a constant phase rotation (i.e., a frequency independent one) that renders the data maximally nonGaussian. The rationale behind the Wiggins algorithm and variants is that convolution of any filter with a time series that is white with respect to all statistical orders renders the outcome more Gaussian. The optimum deconvolution filter is therefore the one that ensures that the deconvolution output is maximally nonGaussian.

I extend the constant-phase rotation method in three ways. (1) I modify it such that it can handle non-stationary (i.e., time-varying) data. (2) I show how the time-varying wavelet can be extracted which can serve as a more familiar quality-control tool for interpreters than phase information alone. (3) I demonstrate how time-varying amplitude and phase deconvolution can be applied by means of Wiener filtering.

## Method

### *Phase estimation*

To estimate the phase of the wavelet, I use a simplification of the blind deconvolution method developed by Wiggins (1978) based on kurtosis maximization. Levy and Oldenburg (1987), Longbottom et al. (1988) and White (1988) greatly reduced the number of degrees of freedom in the phase estimation problem by proposing that a seismic wavelet in the later processing stages can be accurately described by a constant phase approximation, leaving only a single degree of freedom and thereby a robust inversion procedure. The optimum phase is estimated by applying a series of constant phase rotations to the data. The angle corresponding to the maximum kurtosis value determines the most likely wavelet phase.

The rotated trace  $x_{rot}$  is computed from the original trace  $x$  by

$$(1) \quad x_{rot}(t) = (\cos \phi)x(t) + (\sin \phi)H[x(t)],$$

with  $\phi$  the phase rotation angle and  $H[.]$  the Hilbert transform. Note that the rotation angle  $\phi$  can be time-dependent. The most likely phase angle  $\phi_{kurt}$  corresponds to the maximum kurtosis value. This is easiest estimated using a grid search with test angles  $\phi$  between -180 and 180 degrees. The kurtosis is averaged over tens of traces to ensure robustness.

Because of the large reduction in degrees of freedom the described phase estimation method by kurtosis maximization can be extended to cope with time-varying phase changes by subdividing each section into partly overlapping time windows. A single phase is estimated for each window. An overlap of 67% is used such that rapid phase changes indicate that the window size is likely to be too small. The extracted phase is assigned to the center of each analysis window. A linear interpolation is done between each center position to recover the phase variations in between evaluation points. At the start and end times the wavelet phase is assumed to be constant.

If phase-only deconvolution is desired then this can be accomplished by expression 1 as well. In this case the rotation angle  $\phi$  becomes time dependent and is exactly equal to minus the time-varying phase just extracted from the kurtosis analysis, that is,  $-\phi_{kurt}(t)$ .

### *Wavelet estimation*

Wavelet estimation is straightforward once the phase is known. Only the amplitude spectrum is left to be estimated. This is done by (1) averaging the amplitude spectra of all traces in each time window, (2) multiplying the averaged window in the time domain by a Hanning taper for enhanced robustness, while (3) ensuring that the amplitude at the Nyquist frequency remains zero. This procedure leads to a symmetric, zero-phase wavelet. In the frequency domain the final estimated wavelet is then given by

$$(2) \quad W_j(f) = |W_{av,j}(f)| \exp\{i\phi_{kurt,j} \operatorname{sgn}(f)\},$$

with  $|W_{av,j}|$  the averaged amplitude spectrum,  $\phi_{kurt,j}$  the constant phase angle determined by evaluating the kurtosis in window  $j$  and  $\operatorname{sgn}(\cdot)$  the sign function.

### *Time-varying Wiener filtering*

In the deconvolution problem we assume that the observed trace  $x$  is the result of the convolution of the wavelet  $w$  with the reflectivity series  $r$  plus some superposed noise  $n$ . That is,  $x(t) = w(t) * r(t) + n(t)$ , where  $*$  indicates convolution and  $t$  represents time. The objective is to find a filter  $g(t)$  such that the outcome  $y(t)$  is as close as possible to the original reflectivity series  $r(t)$ . Thus,  $y(t) = g(t) * x(t) \approx r(t)$ .

Because of the presence of the noise, the reflectivity series  $r$  cannot be recovered perfectly and a compromise needs to be achieved between noise amplification and successful recovery. If the wavelet  $w(t)$  is known then the time-domain Wiener filter  $g_w(t)$  achieves an optimum solution. In the frequency domain it is given by

$$(3) \quad G_w(f) = \frac{W^*(f)}{|W(f)|^2 + \sigma_n^2},$$

with  $f$  frequency,  $\sigma_n^2$  the noise variance, and  $*$  indicates complex conjugate (Berkhout, 1977).

A specific Wiener filter is created for each individual wavelet  $W_j(f)$  estimated in window  $j$  using expressions 2 and 3. Both the estimated wavelets and all traces are zero-padded such that they have the same length and to ensure that wrap-around effects due to circular convolution in the frequency-domain are prevented. Each individual Wiener filter thus created is applied to all traces in their entirety. Linear interpolation of the resulting deconvolved sections  $y_j(t)$  for each window  $j$  then yields the final deconvolved section  $y(t)$ .

Further details on the implementation, a discussion of the assumptions on which the method is based, and potential quality control measures for evaluating the wavelet estimates and deconvolution results can be found in Van der Baan (2008).

### Real-data example

To illustrate the whole procedure, I use a real data example. Figures 1 and 2 display the data before and after deconvolution, the extracted wavelets, and associated phase and kurtosis variations. Phase variations show a step-like change from -75 to -21 degrees (Figure 2c). The kurtosis variations indicate that the phase is well resolved except possibly for the last wavelet which has a low kurtosis value. The wavelets broaden with time indicating the presence of seismic attenuation.

The original data and the result of phase-only deconvolution are shown in Figure 1. Most variations occur in the shallow most part as can be seen in Figures 1c and 1d which are zoom-ins on the top left most corner. Examples where the phase rotation has rendered the reflectors approximately zero phase can be seen at 0.48s and 0.75 s as indicated by the arrows.

Designing a single deconvolution filter does not produce suitable results for this dataset. For instance, the global wavelet has a phase of -57 degrees and contrasts increasingly from the five extracted local wavelets (Figure 2a). Time-varying wavelet extraction and deconvolution is required for this dataset.

### Conclusions

Phase mismatches occur sometimes between final processed sections and zero-phase synthetics based on well logs, despite best efforts for controlled-phase acquisition and processing. The invoked controlled-phase strategies are generally based on deterministic corrections derived from field measurements and physical laws.

Kurtosis maximization by constant phase rotation is a statistical method that can reveal the phase of a seismic wavelet. It is sufficiently robust that it can even detect time-varying phase changes. Phase-only corrections can then be applied by means of a time-varying phase rotation. Alternatively amplitude and phase deconvolution can be achieved using time-varying Wiener filtering.

A statistical analysis of the data alone cannot reveal whether a remnant phase indicates that the data acquisition and processing strategy was unsuccessful or represents a true geologic feature. It yields nonetheless relevant information about the data which can be used to zero-phase time-varying observations, as a quality control to check deterministic phase corrections, or even as an individual analysis tool.

Time-varying wavelet extraction and deconvolution can also be employed as a robust alternative to amplitude-only inverse-Q deconvolution. The latter tends to be unstable since it attempts often to restore information below the noise level of the data, e.g., amplitudes outside of the passband of the local wavelet. Time-varying wavelet extraction and deconvolution on the other hand seek only to restore amplitudes within the local passband of the wavelet. It is therefore inherently stable.

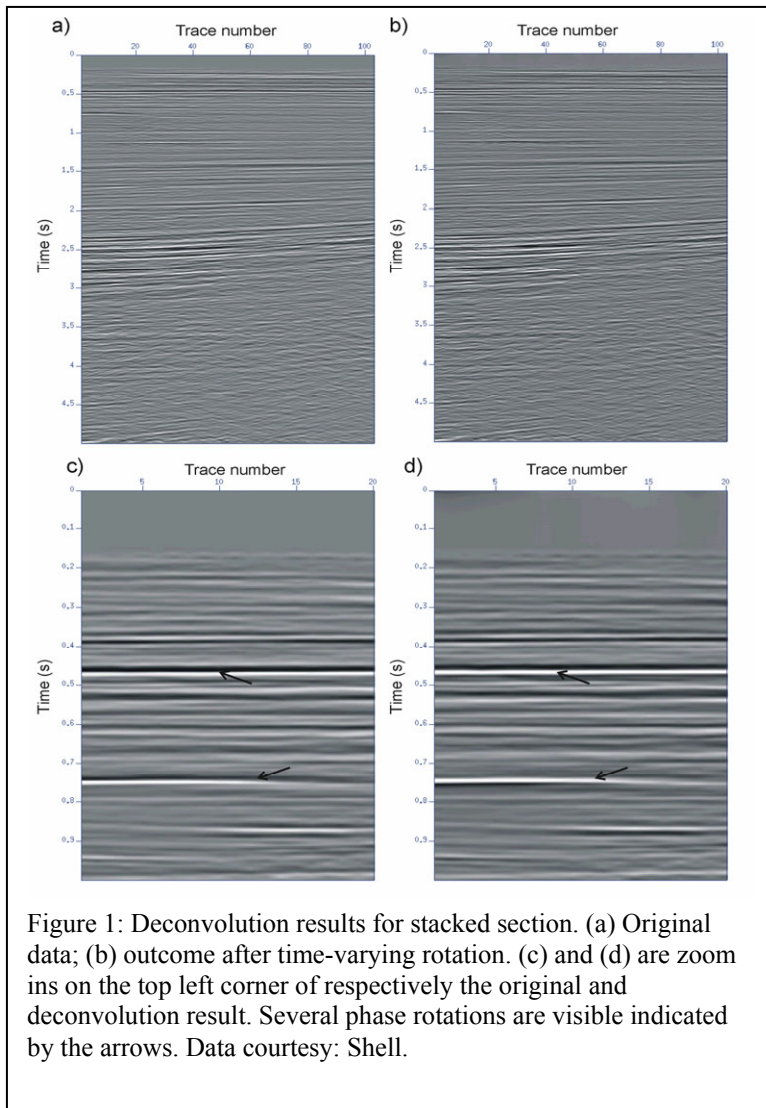


Figure 1: Deconvolution results for stacked section. (a) Original data; (b) outcome after time-varying rotation. (c) and (d) are zoom ins on the top left corner of respectively the original and deconvolution result. Several phase rotations are visible indicated by the arrows. Data courtesy: Shell.

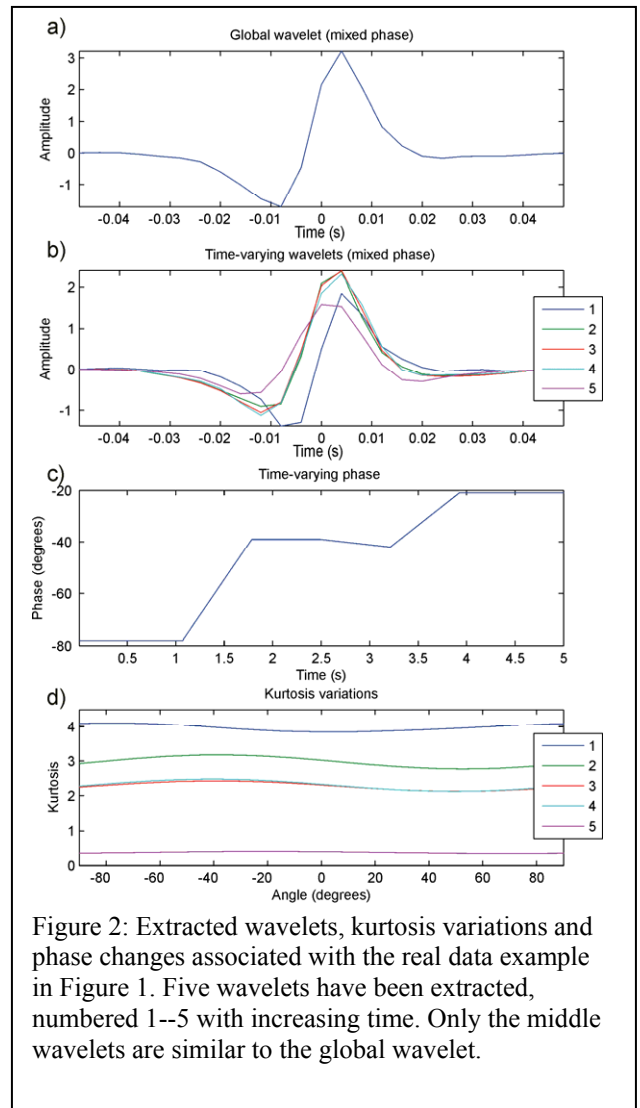


Figure 2: Extracted wavelets, kurtosis variations and phase changes associated with the real data example in Figure 1. Five wavelets have been extracted, numbered 1--5 with increasing time. Only the middle wavelets are similar to the global wavelet.

## Acknowledgements

The author thanks the BG group, BP, Chevron, the Department of Trade and Industry, and Shell for financial support of the project Blind Identification of Seismic Signals, and Shell EP Europe for permission to use the data. The author was an employee of the University of Leeds when the majority of this work was done.

## References

- Berkhout, A. J., 1977, Least-squares inverse filtering and wavelet deconvolution: *Geophysics*, **42**, 1369–1383.
- Levy, S., and D. W. Oldenburg, 1987, Automatic phase correction of common-midpoint stacked data: *Geophysics*, **52**, 51–59.
- Longbottom, J., A. T. Walden, and R. E. White, 1988, Principles and application of maximum kurtosis phase estimation: *Geophysical Prospecting*, **36**, 115–138.
- Trantham, E. C., 1994, Controlled-phase acquisition and processing: 64<sup>th</sup> Annual International Meeting, SEG, Expanded Abstracts, 890–894.
- Van der Baan, M., 2008, Time-varying wavelet estimation and deconvolution by kurtosis maximization: *Geophysics*, **73**, no. 2, V11–V18.
- White, R. E., 1988, Maximum kurtosis phase correction: *Geophysical Journal International*, **95**, 371–389.
- Wiggins, R., 1978, Minimum entropy deconvolution: *Geoprospection*, **16**, 21–35.