# Spherical Wave Model Tests of VSP-Based Q-Estimation Techniques

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## Summary

As a first step toward a better understanding of stratigraphic attenuation of seismic waves, we investigate the response to spherical waves of a simple density step model. The Weyl/Sommerfeld integral is utilized to compute synthetic VSP down-going waves by numerical integration. Q-factors are estimated from these down-going wave fields by applying the spectral ratio method and the analytical signal method. We find that at larger depths, away from density steps, both methods recover the model Q-factor quite well. A departure of the recovered Q from model Q at shallow depths is noticeable for both methods and is thought to be caused by near-field effects. Q-estimation errors for the analytical signal method are found to be considerable in the vicinity of single interfaces investigated. Q-estimates obtained by spectral ratios appear to be less sensitive to step changes in density values, at least in noise-free situations.

## Introduction

Seismic quality factors (Q) have been an active research topic for several decades now and interest appears to be increasing steadily as evidenced by the number of attenuation-related publications and conference presentations. However, there remain practical difficulties in Q-estimation due to restricted bandwidth, poorly constrained multiple scattering, and geometric spreading losses (Best, 2007). It is our aim to eliminate some of these difficulties by exploring the mechanism of stratigraphic attenuation. As a first step toward a better understanding of stratigraphic attenuation, we investigate a simple step model in this study. VSP measurements are commonly made with down-hole receivers and their first arrivals are considered to be multiple free. In fact, an entire time zone for upgoing waves following the first arrivals is said to be largely multiple free (after deconvolution) and *corridor stacks* are computed for this zone. However, transmission effects have modified the amplitude behaviour even of isolated down-going wave fields. What is the sensitivity of Q-factors estimated from first arrival amplitudes with respect to transmission coefficients? In a classic paper by O'Doherty and Anstey (1971) the point is made that forward scattering overpowers the true direct wave in seismic reflection measurements of a layered earth. Richards and Menke (1983) show by numerical experiments that scattering attenuation can be significant and should not be neglected. Mateeva (2003) devotes an entire chapter to "*Distortions In VSP Spectral Ratios*

*Caused By Thin Horizontal Layering".* Plane wave analysis is used there to investigate the influence of geology on spectral ratios. By contrast, spherical waves are employed for this contribution. The Weyl/Sommerfeld integral is utilized to compute synthetic VSP down-going waves by numerical integration. Q-factors are estimated from these down-going wave fields by applying the spectral ratio method and the analytical signal method.

## Theory

The derivation of spectral ratio methods is based on the definition of the quality factor Q (see for example Tonn, 1991). Not unexpectedly then, for a homogeneous medium, far-field Q-factors can be recovered exactly (within numerical accuracy). A change in elastic parameters however can modify transmitted waves (as well as cause reflections) and alter the frequency response which means spectral ratios are also modified. Aki and Richards (1980, p201) give an equation for the transmitted potential of a two-layer acoustic case (Weyl/Sommerfeld integral approach). The generalization for a layered elastic earth (3D equation for a 1D earth, see for example Ewing et al., 1957) can be written as

$$u_p(\omega) = i\omega e^{-i\omega t}\int_0^\infty \prod_{j=2}^{n}\left[\frac{\alpha_j}{\alpha_{j-1}}T_j(p)\right] \times \left[\frac{p^2}{\xi_n}J_1(\omega pr)\sin(i_n) - ipJ_0(\omega pr)\cos(i_n)\right] e^{i\omega\sum_{k=1}^{n}(\Delta z_k \xi_k)}\,dp \qquad (1)$$

where $u_p(\omega)$ is the P-wave displacement along the ray (at the current receiver and at $\omega$), $\omega$ is the frequency in radians, $t$ is the time, $n$ is the number of layers from the source down to the current receiver, $\alpha_j$ is the P-wave velocity in layer $j$, $T_j(p)$ is the Zoeppritz transmission coefficient from layer $j-1$ to layer $j$, $p$ is the horizontal slowness, $\xi_n$ is the vertical slowness of layer $n$, $J_0$ and $J_1$ are zero and first order Bessel functions of the first kind, $r$ is the range (horizontal offset between source and receivers), and $i_n$ is the ray angle in layer $n$ (at the current receiver).

A number of frequency points are computed at every receiver location (depth $z$) according to the desired bandwidth/wavelet. The time-domain wavefield can then be obtained by inverse Fourier transforming. Note that Equation (1) models transmitted waves only and, because of the Bessel functions, spherical spreading is included. Also modelled are near-field and far-field effects.

## The Density Change Model

The parameters for this earth model are α=2000m/s, β=879.88m/s, ρ=2400kg/m$^3$ and $Q_p$=100. Zero-offset synthetic VSP traces computed with Equation (1) are displayed in Figure 1. Amplitude decay with increasing depth is clearly visible. Also note the phase rotation between shallower and deeper traces where deep traces are roughly zero-phase but shallow traces are ~90° phase rotated. As the near-field decays with $1/z^2$ (in contrast to $1/z$ for the far-field) its influence quickly disappears with increasing depth. Figure 2 shows log magnitude spectra as generated with Equation (1) for depths from 15m to 61m. The zero-phase (non-causal) Ormsby wavelet employed for these computations has the parameters 5/15-80\100 Hz. At approximately 580m depth an abrupt density change to A) 1200kg/m$^3$ (half density step) and to B) 4800kg/m$^3$ (twice density step) is introduced. The traces for the density step model of case A) (a density decrease) computed with Equation (1) are plotted in Figure 3 for the vicinity of the density step location at approximately 580m depth. A density decrease implies a decrease in acoustic impedance which leads to an increase in particle displacement. Indeed, close inspection of Figure 3 reveals an amplitude increase at depths exceeding the density step location at about 580m. Superimposed at all depths is the amplitude decay caused by spherical spreading. The move out between traces is linear (there is no variation with depth) because of constant velocities. Note the ringing typical for Ormsby wavelets. Figure 4 is the equivalent to Figure 3 computed for case B) (a density increase). A density increase results in an acoustic impedance increase which means particle displacement is decreased. As expected, maximum trace amplitudes in Figure 4 are decreasing at depths exceeding the density step location at about 580m. Amplitude decay because of spherical spreading is present but barely visible. Figures 3 and 4 are plotted at different scales to highlight trace-to-trace amplitude changes.

**Density Change and the Spectral Ratio Method**

Next, the spectral ratio method or SRM (Tonn, 1991) is applied to the density change models introduced in the previous section. Figure 5 shows the Q-factors estimated from the computed model traces. These Q($z$) curves depend on smoother lengths and depth intervals used for estimation with the spectral ratio method. In this case the amplitude spectra of three neighbouring traces are averaged and the estimation depth interval is $\Delta z$=38m. Away from the density step the model Q-factor of 100 is quite well recovered in Figure 5. The departure of recovered Q from model Q at depths shallower than approximately 200m in all Q($z$) displays is thought to be caused by near-field effects (see Figure 2 for the slope behaviour of near-field log magnitude spectra). At the density step decrease in Figure 5 SRM is seen to firstly underestimate Q and then secondly overestimate Q as the analysis window slides across the discontinuity. For a density step increase, the situation is reversed.

**Density Change and the Analytical Signal Method**

The analytical signal method (ASM) estimates the quality factor (Q) from the decay of instantaneous amplitudes with depth (see Tonn, 1991). The ASM-technique (time-domain) is considerably more sensitive to amplitude disturbances than the SRM-approach (frequency-domain). Therefore, for ASM-testing, we reduce the density step size to ±5% (and reduce the sample interval to 1/8ms). Q-estimates for model VSPs with a density step to 95% and a density step to 105% are shown in Figure 6. Note the density step location is at approximately 580m depth. For a decrease in density, Q is overestimated and vice versa. There is a Q-estimation error for the entire depth-range spanned by the combination of smoother length and analysis window. Even though the far-field Q-estimate closely approaches the model Q-factor of 100, the near-field influence on the Q-estimate extends further in depth for the ASM-algorithm than it does for the SRM-technique.

**Conclusions**

Away from density steps, at larger depths, all methods recover the model Q-factor of 100 quite well. The departure of the recovered Q from the model Q at shallow depths is noticeable for all methods and is thought to be caused by near-field effects. Where the near-field predominates, we can expect time-domain Q-estimation methods to *underestimate* Q because of the $1/z^2$ amplitude decay. The specific slope behaviour of magnitude spectra observed for decreasing depths leads to *overestimated* Q for the spectral ratio method and even to negative Q near the source. When estimating Q from actual VSP data (Haase and Stewart, 2006; ibid, 2007) the same kind of *near-field behaviour* is observed. Q-estimates obtained by the spectral ratio method appear to be least sensitive to step changes in density values, at least in noise-free situations. For the analytical signal method density-step responses of Q($z$) are somewhat smeared out because of depth averaging.

## Acknowledgements

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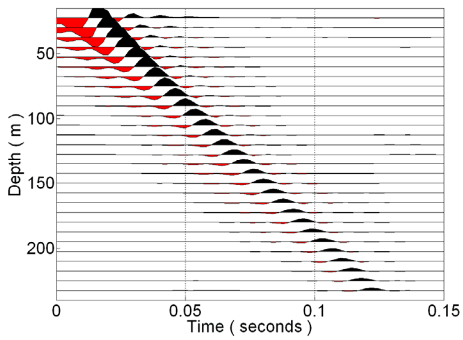


Figure1: Zero-offset synthetic VSP computed with a zero-phase (non-causal) 5/15-80\100Hz Ormsby wavelet.

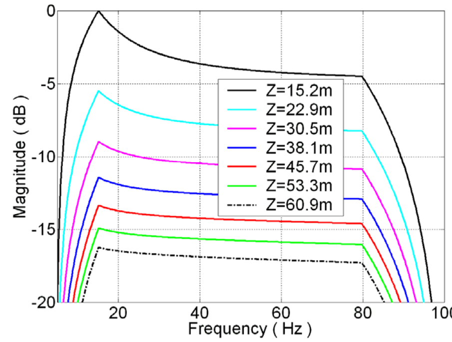


Figure2: Near-field magnitude spectrum for depth range from 15m to 61m.

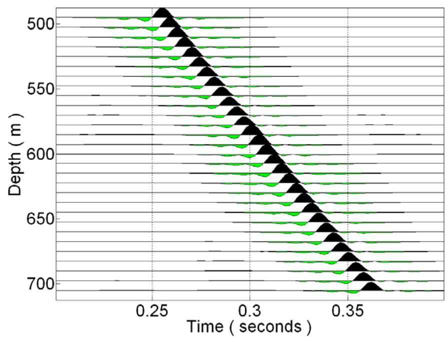


Figure3: Selected traces of half-density-step model.

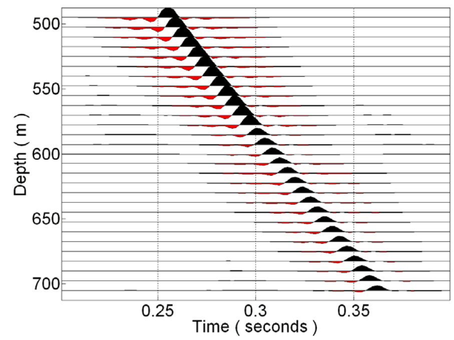


Figure4: Selected traces of twice-density-step model.

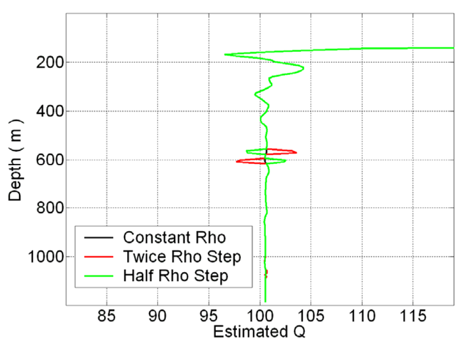


Figure5: Q-estimate by spectral ratio method (density step model).

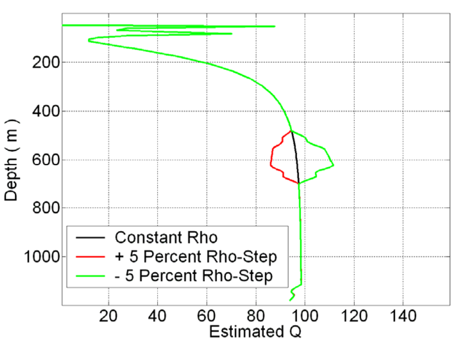


Figure6: ASM Q-estimate of +/- 5% density step model (1/8 ms sample interval).