Potential Field and Gradient Component Transformations and Downward Continuation: New Space Domain Techniques and their Applications

Bingzhu Wang*
Cameco Corporation, Saskatoon, SK, Canada
Bingzhu_Wang@Cameco.com

Brian Powell
Cameco Corporation, Saskatoon, SK, Canada

Edward S. Krebes
University of Calgary, Calgary, AB, Canada

and

Dhananjay Ravat
Southern Illinois University, Carbondale, IL, United States

Introduction

Potential field and gradient transformations facilitate geologic interpretation by transforming the observed data into various new forms. Direct measurements are sometimes either inaccurate, or expensive, or even impossible. The popular FFT transformation algorithms are prone to noise and require uniform data spacing.

When the computational domain is large enough, the infinite double integrals for component transformations can be approximated by definite double integrals and solved by the spline technique. The downward continuation is accomplished by solving Fredholm integral equations of the first kind using the spline bases.

Techniques and Applications

1. Component Transformations

Potential field and gradient component transformations are necessary to meet the needs of many data enhancement, direct interpretation, and inversion techniques in which the required components are either not measured or are inaccurate. We briefly present the theory here.

Let $U(x,y,z)$ be the potential, $\mathbf{f}(x,y,z)$ be the potential field, $\Delta T(x,y,z)$ be the total-field magnetic anomaly, and $\boldsymbol{D}_{ij}(x,y,z)$ be the potential-field gradient tensor. For the 3D case, only five of the nine components of the gradient tensor are independent. Integral relations on a horizontal surface between the vertical component of a potential field or gradient and the horizontal components of the potential field or gradient from arbitrary sources are

$$V_z(\xi,\eta) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(\xi - x)V_x(x,y) + (\eta - y)V_y(x,y)}{[(x-\xi)^2 + (y-\eta)^2]^{3/2}}dxdy \quad , \qquad (1)$$

$$V_x(\xi,\eta) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(\xi - x)V_z(x,y)}{[(x-\xi)^2 + (y-\eta)^2]^{3/2}}dxdy \quad , \qquad (2)$$

$$V_y(\xi,\eta) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(\eta - y)V_z(x,y)}{[(x-\xi)^2 + (y-\eta)^2]^{3/2}}dxdy \quad . \qquad (3)$$

where V_z can be f_z, $\frac{\partial \Delta T}{\partial z}$, or D_{iz}; V_x can be f_x, $\frac{\partial \Delta T}{\partial x}$, or D_{ix}; and V_y can be f_y, $\frac{\partial \Delta T}{\partial y}$, or D_{iy}; $i = x, y, z.$

When the computational domain is large enough, each infinite double integral can be approximated by a definite double integral and solved by the spline technique (Wang, 2006). For example,

$$D_{zz}(\xi,\eta) = \frac{1}{2\pi}\sum_{i=-1}^{Nx+1}\sum_{j=-1}^{Ny+1}[N_i^{-1}(b) - N_i^{-1}(a)]C_{i,j}[N_j^{-1}(d) - N_j^{-1}(c)] \quad . \qquad (4)$$

2. Downward Continuation

Downward continuation transforms the potential field and gradient measured on one surface to the field that would be measured on another surface closer to the sources. Although it is an inherent ill-posed inverse problem, downward continuation can relatively enhance the effects of shallower, smaller sources. It can also increase the resolution and provide useful information for further interpretation. When it is necessary to merge or compare potential field or gradient data measured at different altitudes, downward continuation provides a way to transform individual surveys onto a common surface.

The new downward continuation technique is accomplished by solving Fredholm integral equations of the first kind in the space domain using the spline bases. The technique is robust and can stably downward continue data over much larger distances than can the standard FFT technique.

3. Applications

We present examples to show how the spline-based component transformations can verify the quality of real potential field or gradient data, and can meet the needs of direct interpretation and inversion techniques. We also show downward continuation examples using airborne potential field or gradient data and compare them with ground data.

Conclusions

The results of spline-based component transformations and downward continuation of potential field and gradient data, both synthetic and real, show that the spline-based algorithms are accurate, noise-tolerant and practical. Data spacing can be either regular or irregular. Comparisons with theoretical data show that the spline-based algorithms are significantly more accurate and less noisy than the FFT approaches; the spline-based techniques have higher resolution and smaller edge effects than the FFT approaches. Thus, the spline techniques are likely to lead to a better interpretation and understanding of the sources of the potential field and gradient than the FFT techniques.

References

Wang, Bingzhu, 2006, 2D and 3D potential-field upward continuation using splines: Geophys. Prospect., **54**, 199-209.