Traveltime Computations for Locating the Source of Micro Seismic Events and for Forming Gridded Traveltime Maps

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A new method is presented for estimating the source location of an event, based on the assumption that the event creates a spherical wavefront which is recorded by a number of receivers with fixed locations. This method is suitable for locating microseismic events, mapping traveltimes that are computed along raypaths to gridded times, or for propagating first arrival times on a grid.

Previous methods that computed traveltimes with spherical (or circular) wavefront assumptions used hyperbolic equations to estimate the location of the source. This method is based on the tangency of spheres (or circles) with radii defined by clock-times. Straightforward analytic solutions exist with two possible solutions, with the correct solution easily identified.

Introduction

Well fraccing applications require estimating the location of microseismic events that occur during the fraccing process. Receivers at fixed locations record the clock-times of events, and these traveltimes are then used to estimate the location and time of the source. Knowledge of the location of these events help define the volumetric extent of the fraccing process. The velocity is assumed to be constant between the source and receiver locations.

In a similar application, unstable geological areas may be monitored with a number of receivers that record microseismic events caused by internal stress. The location and number of events may indicate an imminent land slide or earthquake.

Kirchhoff migrations require the traveltimes from sources and receivers to define diffraction shapes. These traveltimes may be computed along raypaths and then transferred to a traveltime grid. The raytracing method provides flexibility as it offers a choice between first arrival times, the arrival time of maximum energy, or a combination with multiple arrivals for rays that take different paths. Traveltimes along the rays are mapped to gridded times that are then used for migration.

Traveltimes may also be computed directly on a grid. This method typically uses the known traveltimes on three corners of a square to estimate the traveltime on the remaining corner. An industry standard (Vidale 1988) uses a finite difference solution to the eikonal equation. This method assumes plane waves and produces significant errors when the radius of curvature of the wavefront approaches the grid size (Bancroft 2005). The new spherical method competes with hyperbolic assumptions (Vidale 1988, Bancroft 2005) as both assume a spherical wavefront.

The raypath and gridded traveltime estimations assume that the local velocity in the area defined by the defined times is constant. This area may be extended, without error, to cover an area that includes the apparent source location.

Two dimensional (2D) applications assume three known (receiver) locations, while three dimensional applications require four known locations.

Theory

The method presented in this paper assumes the wavefront is locally spherical in the area of interest, and then computes the center of curvature which becomes the apparent source of the spherical wavefront. The excitation clock-time at the source point is also computed.

The solution is based on the tangency of circles for 2D data and spheres for 3D data. We develop the method for 2D data as it can be visualized and then extend the method to 3D using parallel development.

We start by assuming a source location (x_0, z_0) and three arbitrarily located receiver points (x_1, z_1), (x_2, z_2), and (x_3, z_3) as illustrated in Figure 1a. The traveltimes recorded at these locations are t_1, t_2, and t_3 and are clock-times, i.e. not the travel times from the source point. We assume the event started at the source location with time t_0. The traveltimes between the source and receiver locations (delta-times) are then defined by $t_{01} = (t_1 - t_0)$, $t_{02} = (t_2 - t_0)$, and $t_{03} = (t_3 - t_0)$. These delta-times may be used to define radii r_{01}, r_{02}, and r_{03} or distances from the source to the receiver locations. Rather than draw circles centered at the source point, we draw circles centered at the receiver locations as illustrated in Figure 1b. The intersection of these circles define the source location. However, we do not know these radii at the beginning of our problem.

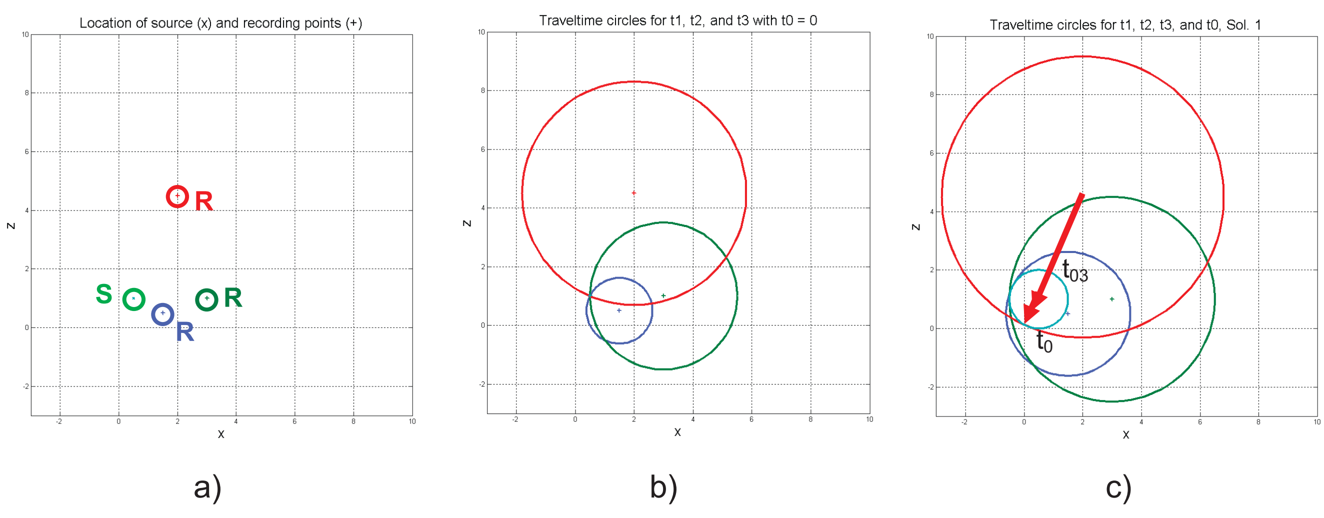


Figure 1. a) The geometry of a source (S) location and three receiver (R) locations, b) the delta-time (initially unknown) radius from the receivers intersecting at the source location, and c) clock times that are tangent to a circle with radius of the source time centered at the source location.

We do know the times t_1, t_2, and t_3 and we can draw circles representing their radii as illustrated in Figure 1c. The radii of these circles is the sum of the source time t_0 plus the delta-times to the receiver, i.e. $r_1 = v(t_0 + t_{01})$, $r_2 = v(t_0 + t_{02})$, and $r_3 = v(t_0 + t_{03})$. It is now important to note that these three circles pass beyond the source point with the same distance $v*t_0$ as illustrated in Figure 1c.

Figure 1c also contains a cyan circle about the source point with radius $v*t_0$. This circle is tangent to the three other circles. The points of tangency are on lines drawn from the receiver location and through the source location. At this point in our development we don't know the location (x_0, z_0) or time t_0 of the source point. What we do know is the radius of the three circles in Figure 1c, and that the source location can be found by locating a circle (cyan in Figure 1c) that is tangent to these three known circles.

The problem of finding a circle that is tangent to three given circles has been of interest for centuries and the first solution is attributed to Apollonius of Perga who was born in 261 BC. Geometric constructions are possible but complex, illustrating that a solution should "involve nothing more than quadratic equations", (Website 1). Numerous algebraic solutions are also available on the web, and we will follow one based on Website 2.

For 2D applications we start with equations representing the three circles with radii from the source to the three receiver locations, i.e.

$$
\begin{aligned}
(x_1 - x_0)^2 + (z_1 - z_0)^2 &= v^2 (t_1 - t_0)^2 \\
(x_2 - x_0)^2 + (z_2 - z_0)^2 &= v^2 (t_2 - t_0)^2 \\
(x_3 - x_0)^2 + (z_3 - z_0)^2 &= v^2 (t_3 - t_0)^2 .
\end{aligned}
$$

We then use algebra (Bancroft and Du 2006a, b) to solve for the unknowns x_0, z_0 and t_0. We use a similar set of equations for the 3D application

$$
\begin{aligned}
(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 &= v^2 (t_1 - t_0)^2 \\
(x_2 - x_0)^2 + (y_2 - y_0)^2 + (z_2 - z_0)^2 &= v^2 (t_2 - t_0)^2 \\
(x_3 - x_0)^2 + (y_3 - y_0)^2 + (z_3 - z_0)^2 &= v^2 (t_3 - t_0)^2 \\
(x_4 - x_0)^2 + (y_4 - y_0)^2 + (z_4 - z_0)^2 &= v^2 (t_4 - t_0)^2 ,
\end{aligned}
$$

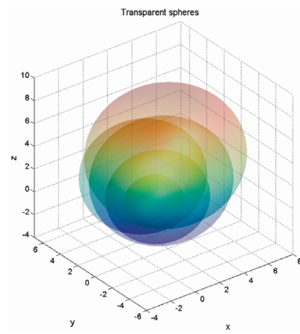
to solve for the unknowns x_0, y_0, z_0, and t_0. The 3D solution still has only two possible solutions.

Comments

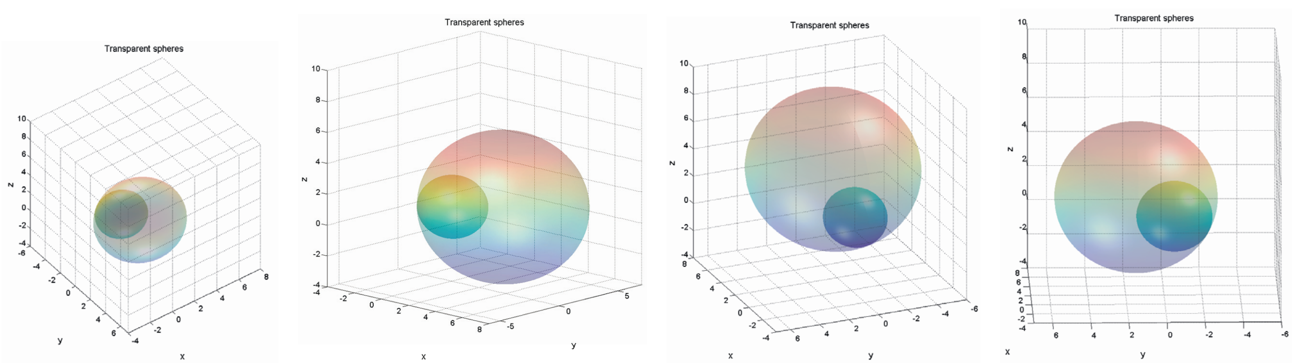
The clock times are abitrary in that a constant time can be added or subtracted without affecting the problem. Consequently, one receiver time can be assumed to be zero and subtracted from all the other clock-times. In this case, the source clock time will become negative. There will be a corresponding slight simplification to the solution.

Observations of the spheres (circles) verify that the source sphere (circle) has a radius less than all the other receiver spheres (circles). This is also intuitively obvious as the source clock-time must be less that that of the clock-times of the receivers as they must include the additional delta-times. This simplifies the choice between the two solutions.

The solutions include one that is internal to all the spheres or external to all the spheres. The correct solution is the one internal to the spheres and will have a radius that is less than the external sphere. These results are illustrated in Figure 2, which contains in (a) one perspective view of all the transparent spheres, with (b), (c), (d) and (e) showing various rotations of each volume to illustrate the tangency of the source sphere with each receiver sphere for a given problem.



a)



b) c) d) e)

Figure 2. Transparent spheres with a) containing the source and all four receiver spheres, then b), c), d), and e) with each containing one receiver sphere together with the source sphere, and with the volumes rotated to illustrate the areas of tangency between the spheres.

Conclusions

A method is presented that computes the center of curvature from a spherical wavefront. Applications of this method are used to locating microseismic events or in computing traveltime maps for a Kirchhoff depth migration.

References

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Website 1, htpp://www.mathpages.com/home/kmath113.htm

Website 2, htpp://mathworld.wolfram.com/ApolloniusProblem.html

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