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# The Heterogeneity Cube: a Family of Seismic Attributes

## Introduction

The proposed seismic attributes measure the average medium heterogeneity from a 3-D seismic datacube. The basic idea behind the heterogeneity cube is that small-scale heterogeneity causes a small-scale footprint on seismic data, whose statistics relate to the statistics of the heterogeneity. Parameterizing the footprint statistics, one obtains a set of seismic attributes which are interpretable as acquisition and processing footprints, stratigraphic or lithologic heterogeneity, or structural heterogeneity. Acquisition and processing footprints may be removable as described by Marfurt et al. (1998). From the stratigraphic viewpoint, the parameters may denote average dimensions and orientations of small sedimentary bodies (Imhof and Toksöz, 2000), while the parameters might relate to average size, spacing, and orientations of fractures and joints for the structural point of view.

The heterogeneity cube nicely complements the coherency cube (Bahorich and Farmer, 1995). The low-pass heterogeneity cube measures the average fluctuation of the seismic signal within a small volume, while the high-pass coherency cube detects subtle changes in the signal, e.g., when crossing faults or facies.

The proposed heterogeneity attributes aid the interpretation of seismic data and provide novel information for reservoir characterization and model building.

## Method

The heterogeneity attributes are volume attributes, i.e., they are calculated for every point $(x, y, z)$ of a seismic poststack datacube $d$. A little probe volume $w$, centered at the current $(x, y, z)$, is extracted from the full datacube $d$. The probe $v$ is then correlated with the datacube $d$ to estimate the local crosscorrelation function ($LCCF$) $\hat{R}(\Delta x, \Delta y, \Delta z; x, y, z)$ for a number of different correlation lags $\Delta x$, $\Delta y$, and $\Delta z$. The $LCCF$ $\hat{R}$ is normalized to unity for $\Delta x = \Delta y = \Delta z = 0$.

$$\hat{R}(\Delta x, \Delta y, \Delta z; x, y, z) = \frac{\rho(\Delta x, \Delta y, \Delta z; x, y, z)}{\rho(0, 0, 0; x, y, z)}$$

The unnormalized $LCCF$ $\rho(\Delta x, \Delta y, \Delta z; x, y, z)$ is defined as

$$\rho(\Delta x, \Delta y, \Delta z; x, y, z) = \frac{1}{N} \sum_{\delta x, \delta y, \delta z} w(x + \delta x, y + \delta y, z + \delta z) \cdot d(x + \delta x + \Delta x, y + \delta y + \Delta y, z + \delta z + \Delta z)\,,$$

where $N$ is the number of non-zero terms in the summation. The resulting local $LCCF$ $\hat{R}(\Delta x, \Delta y, \Delta z; x, y, z)$, however, contains too many values to be of direct use even if only a few lags were calculated. To be useful as seismic attributes, the number of values needs to be reduced. Instead of directly using the raw estimate $\hat{R}$ of the $LCCF$, the number of parameters is decimated by fitting the estimate $\hat{R}$ with a model $LCCF$ $\bar{R}$ which contains only six free parameters.

Presently, the model $LCCF$ $\bar{R}$ is Gaussian in the horizontal directions, but exponential in the vertical. This choice of model yields greater continuity and smoothness in the lateral directions, but more variability and roughness in the vertical. Each direction is scaled independently with a characteristic length: $a > b > c$. The larger lengths, $a$ and $b$, are used for the horizontal Gaussian model, while the shortest length $c$ is used in the vertical exponential model. For greater flexibility, the model *LCCF* $\bar{R}$ is rotated by $\phi_z$, $\phi_y$, and $\phi_x$ around the Cartesian x, y, and z axes:

$$\bar{R}(\Delta x, \Delta y, \Delta z; a, b, c, \phi_x, \phi_y, \phi_z) = r(u, v, w; a, b, c) \quad \text{where}$$

$$r(u, v, w; a, b, c) = \exp\left(-u^2/a^2 - v^2/b^2 - |w/c|\right) \quad \text{and}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \cos\phi_y \cos\phi_z & -\cos\phi_y \sin\phi_z & -\sin\phi_y \\ -\sin\phi_x \sin\phi_y \cos\phi_z + \cos\phi_x \sin\phi_z & \sin\phi_x \sin\phi_y \sin\phi_z + \cos\phi_x \cos\phi_z & -\sin\phi_x \cos\phi_z \\ \cos\phi_x \sin\phi_y \cos\phi_z + \sin\phi_x \sin\phi_z & -\cos\phi_x \sin\phi_y \sin\phi_z + \sin\phi_x \cos\phi_z & cos\phi_x \cos\phi_y \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}.$$

The angle $\phi_z$ denotes the orientation of the largest correlation length $a$, i.e., the direction of maximal continuity. The angle $\phi_y$ specifies the dip of the *LCCF* at the direction of maximal continuity. Finally, the tilt $\phi_x$ indicates how much the *LCCF* has been rotated around the direction of maximal continuity.

The optimal set of parameters $(a, b, c, \phi_x, \phi_y, \phi_z)$ minimizes the root-mean-square difference $\epsilon^2$ between the model *LCCF* $\bar{R}(\Delta x, \Delta y, \Delta z)$ and the data *LCCF* $\hat{R}(\Delta x, \Delta y, \Delta z)$.

$$\epsilon^2 = \sum_{\Delta x, \Delta y, \Delta z} \left(\hat{R}(\Delta x, \Delta y, \Delta z; x, y, z) - \bar{R}(\Delta x, \Delta y, \Delta z; a, b, c, \phi_x, \phi_y, \phi_z)\right)^2$$

The optimal set $(a, b, c, \phi_x, \phi_y, \phi_z)$ is presently determined by systematically scanning of the model space (Imhof and Toksöz, 2000). This approach is very robust, but has a limited resolution due to the tremendous computational costs of testing thousands of different parameter sets. In the future, we will use a non-linear optimization algorithm which even allows to set bounds for the parameters (Zhu et al., 1997). Non-linear optimization yields higher resolution at lower computational costs. The resulting heterogeneity cubes, however, are noisier than the ones obtained by systematic search and often need to be postprocessed with a median filter to remove outliers caused by trapping in local minima or non-conversion.

An optimal set of parameters is found for every point of the seismic datacube $d$. Hence, one obtains seven new datacubes from the scale parameters $a$, $b$, and $c$, the angles $\phi_x$, $\phi_y$, and $\phi_z$, and the minimal misfit $\epsilon^2$ which describes how well the model *LCCF* $\bar{R}$ describes the data.

**Example**

The attributes are calculated from the 3-D poststack datacube from the Stratton field in south Texas (e.g., Hardage et al., 1994). Inline section 50 and two timeslices at 1.0 and 2.0 s are shown in Figure 1. Both upper and middle Frio are undisturbed, while the lower Frio and the Vicksburg are severely faulted. Hence, slices through the heterogeneity cubes at 1.0 and 2.0 s will be very different. The little probe volume spans 19 traces by 19 traces by 19 samples, or 990 ft by 990 ft by 36 ms. To illustrate the results, the same inline section and timeslices are extracted from the heterogeneity cubes. Since the seismic dataset has only been time migrated, dip and tilt are pseudo angles and will need to be mapped to real angles.

Figures 2 to 4 illustrate how the scale parameters $a$, $b$, and $c$ vary inside the datacube, while Figures 5 to 7 map the angles $\phi_z$ (orientation), $\phi_y$ (dip), and $\phi_x$ (tilt). The attributes clearly discriminate the undisturbed upper and middle Frio from the faulted lower Frio and the underlying Vicksburg. Both dip $\phi_y$ and tilt $\phi_x$ are nearly zero for the undisturbed zones, but vary strongly in the deformed areas. Undisturbed areas typically have longer characteristic lengths $a$ and $b$ than deformed ones. Finally, the angles are easy to interpret on timeslices, but they are difficult to visualize on inline sections.

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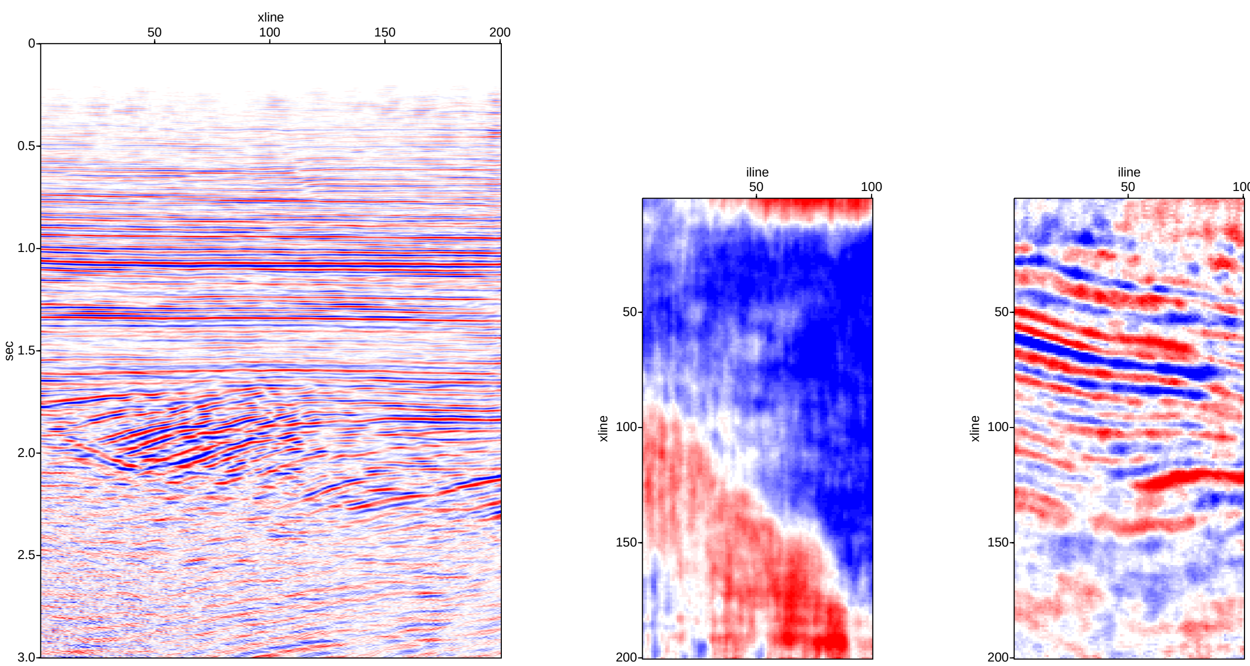


Figure 1: Seismic amplitudes for inline section 50 and timeslices at 1.0 and 2.0 s. The flat Frio reflections overlay the severely faulted Vicksburg formation.

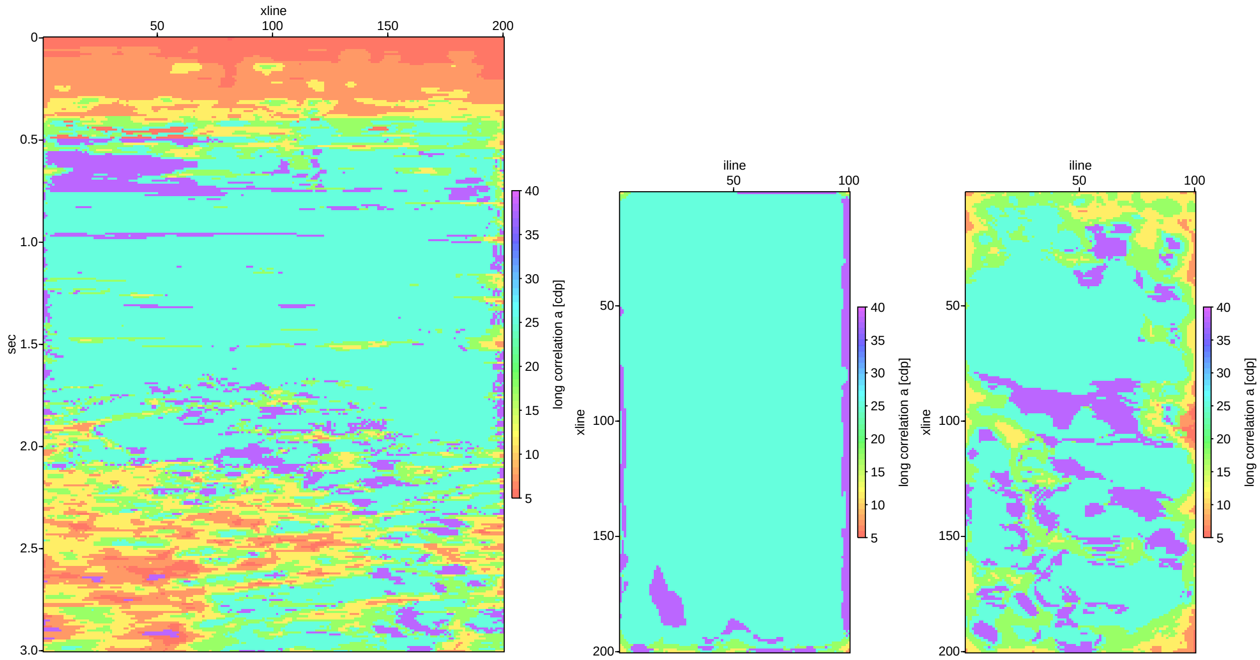


Figure 2: Heterogeneity parameters: maximal correlation length $a$ for inline and timeslices.

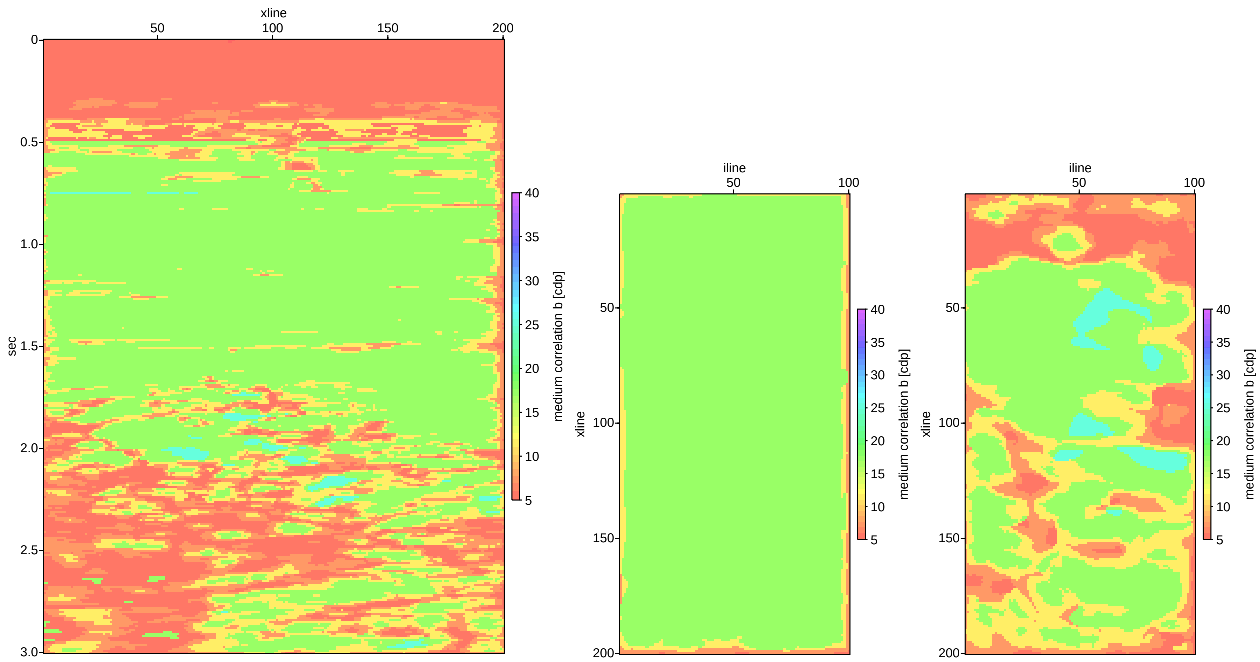


Figure 3: Heterogeneity parameters: intermediate correlation length $b$ for inline and timeslices.

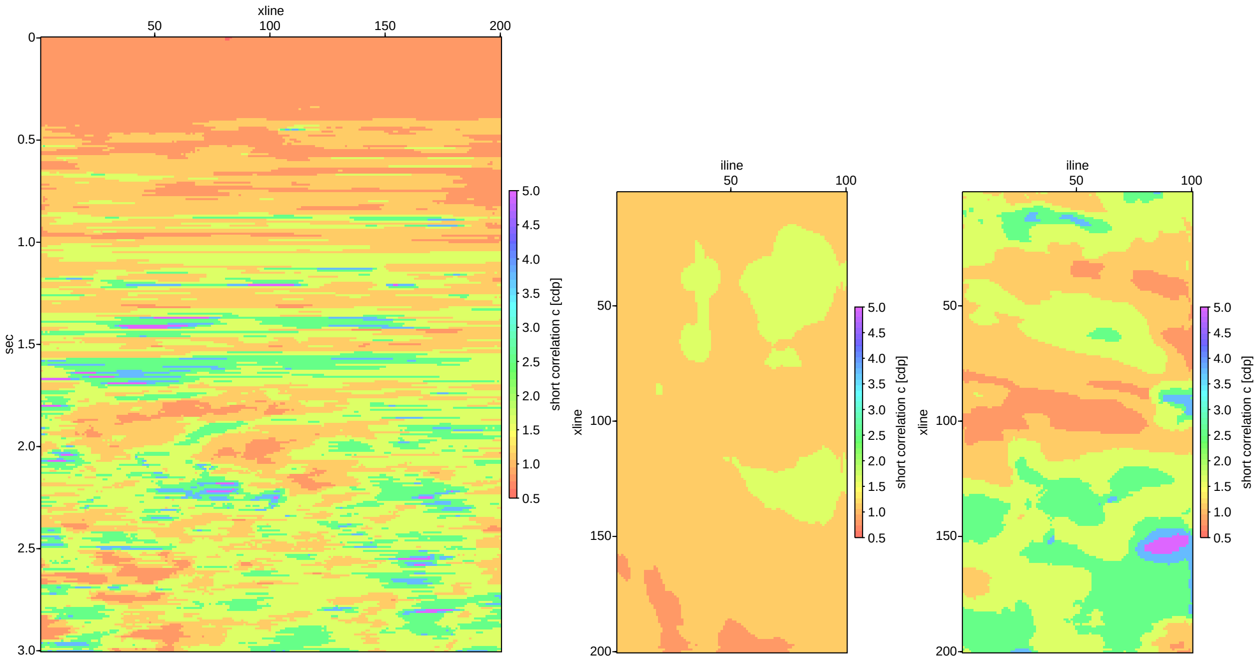


Figure 4: Heterogeneity parameters: short correlation length $c$ for inline and timeslices.

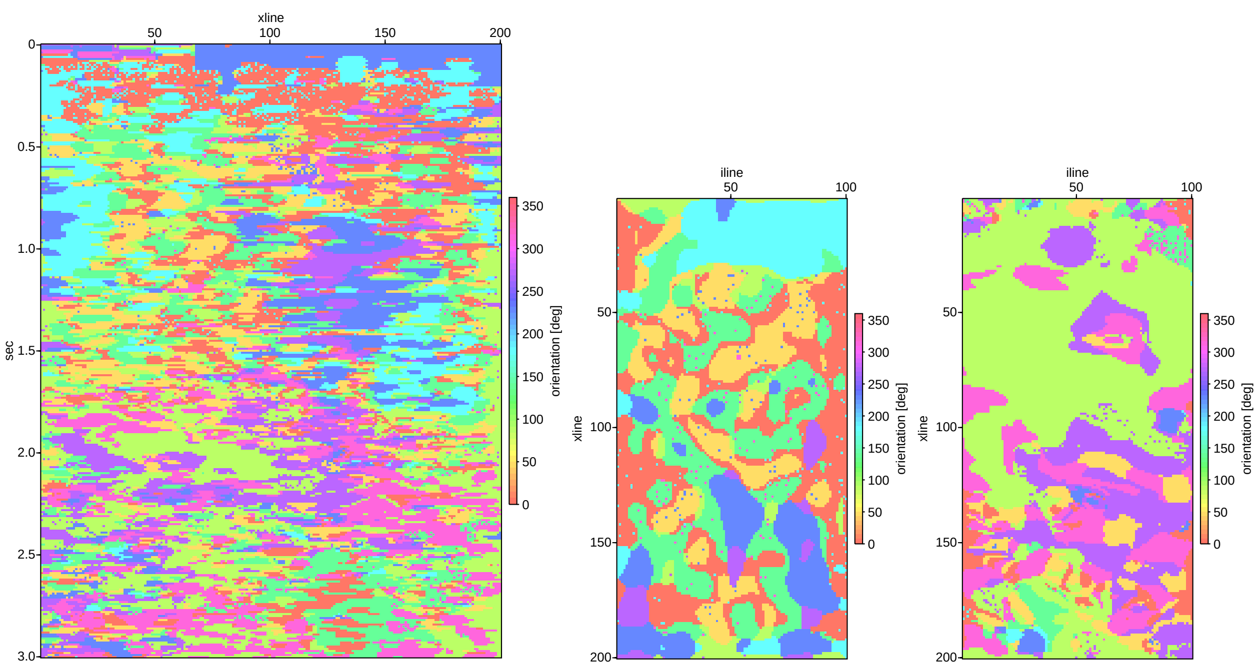


Figure 5: Heterogeneity parameters: orientation $\phi_z$ for inline and timeslices.

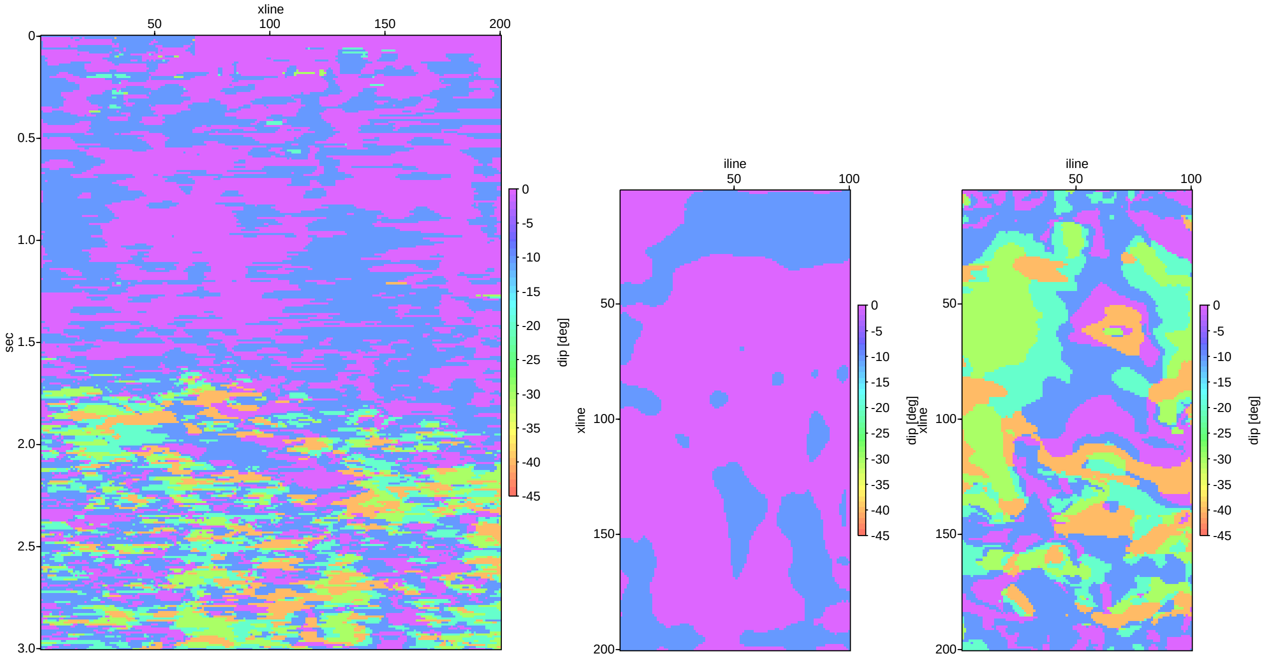


Figure 6: Heterogeneity parameters: dip $\phi_y$ for inline and timeslices.

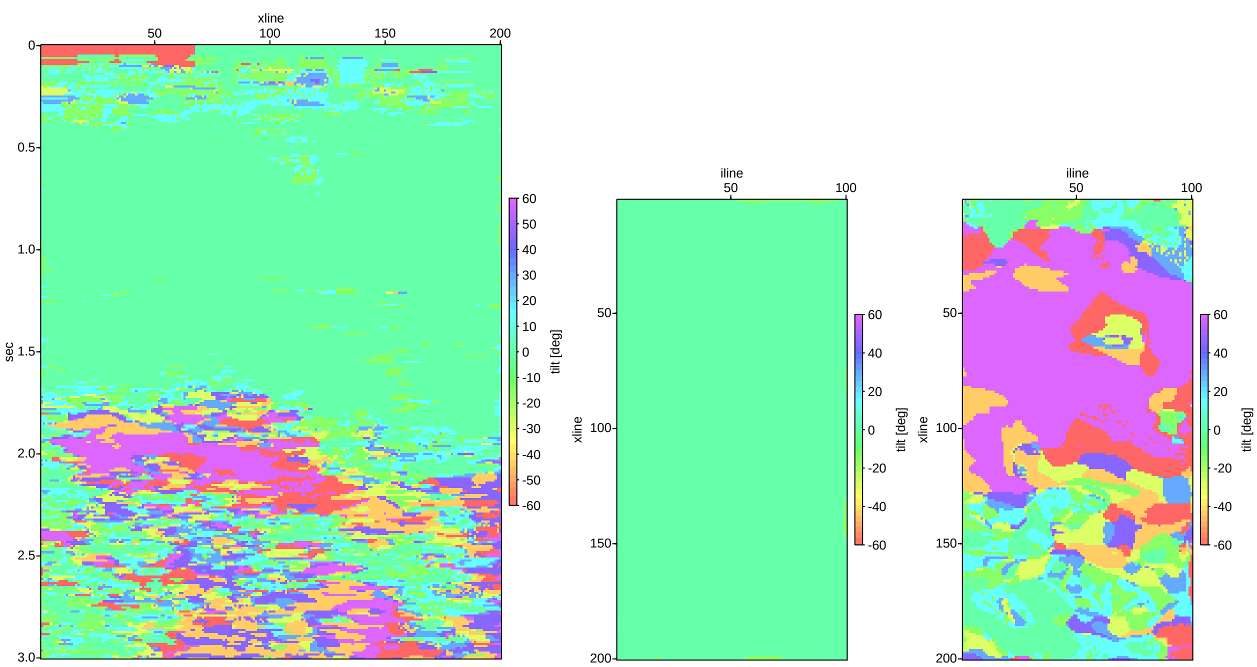


Figure 7: Heterogeneity parameters: tilt $\phi_x$ for inline and timeslices.