Workflow for Zoeppritz AVO Inversion to Estimate Seismic Anisotropy, Geomechanical Properties and TOC of Shale: Case Study of Avalon Shale, Delaware Basin*

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Abstract

Reliable estimation of geomechanical properties (i.e. Young's modulus and Poisson's ratio) and total organic carbon (TOC) content of shale provides important constraints to guide petroleum production by identifying abundant organic sweet spots and locations for effective hydraulic fracturing. However, most conventional amplitude variation with offset (AVO) inversions cannot properly estimate the properties, since the inversions are developed based on linear approximations of Zoeppritz equations assuming weak contrasts and seismic isotropy. Organic-rich shale formations are, however, often anisotropic. In order to overcome limitations of the conventional methods, we propose a workflow to estimate seismic anisotropy and geomechanical properties of organic-rich shale. It is based on analyses of an AVO inversion with full Zoeppritz solutions mainly for P-wave reflection amplitudes.

The anisotropy of the model shale is related to the kerogen volume fraction values using measured well logs and laboratory data for the Avalon Shale in the Delaware Basin. By applying inversion tests, we determine behaviors of the AVO inversion solutions developed for isotropic media when the target shale formation instead has seismic anisotropy related to organic content. These tests show that the inversion accurately determines horizontal P-wave and S-wave velocities and underestimates density when a far angle range is applied with input data. When the angle range is small, the inversion can obtain reliable vertical velocities, and correct density. Therefore, seismic anisotropy of the model can be estimated by comparing these inverted horizontal and vertical velocities. In addition, geomechanical properties of the model are also reliably determined in both
horizontal and vertical directions. We also estimate organic carbon content of the Avalon Shale in Delaware Basin from density values obtained by an amplitude variation with offset (AVO) inversion. The estimation is based on an empirical relationship between kerogen volume fraction and density of the shale.

**Selected References**


Workflow for Zoeppritz PP-AVO inversion to estimate seismic anisotropy, geomechanical properties, and TOC of shale: Case study of Avalon shale, Delaware Basin

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May 20, 2019

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1. Research Objective

2. Method: Nonlinear AVO inversion based on Zoeppritz equations

3. Case Study: Avalon Shale

4. Conclusions
Research Objective
Research Objective and Contributions

- **Research Objective**
  - Completion quality of unconventional shale reservoir

- **Method**
  - Nonlinear Zoeppritz AVO inversion
  - Workflow based on inversion results
  - Empirical relationship between $\rho$ and TOC

- **Contributions**
  - Seismic anisotropy
  - Geomechanical properties (e.g., $E$, and $\nu$)
  - Organic abundance (i.e. TOC) of shale

A Scanning Electron Microscope (SEM) image of shale, provided by Dr. Gibson
Method: Nonlinear AVO inversion based on Zoeppritz equations
Amplitude Variation with Offset (AVO) inversion

- Amplitude Variation with Offset (AVO) inversion
  - to estimate elastic properties of target layers (e.g., Al, Sl, $V_P$, $V_S$, and $\rho$) [Castagna and Backus, 1993].
  - by minimizing the error between observed and modeled AVOs

\[
E(x) = \frac{1}{2} \sum_{i=1}^{N_{\text{obs}}} \|R^c_i(x) - R^m_i\|^2, \tag{1}
\]

where
- $R^m_i$: observed reflectivity at $\theta_i$
- $R^c_i$: modeled reflectivity at $\theta_i$
- $N_{\text{obs}}$: number of observations ($\theta_i$)
- $x$: model parameters, $V_{P1}$, $V_{P2}$, $V_{S1}$, $V_{S2}$, $\rho_1$, and $\rho_2$
Research Motivation: Limitation of conventional AVO methods

- Conventional AVO methods: linearized approximations of Zoeppritz eq.
- Weak contrast & Isotropy
- cf) [Rüger, 1997] (Weak anisotropy & Weak contrast)

ex1: Isotropy & Strong contrast  
ex2: Strong anisotropy & Weak contrast

- How to overcome? Using Zoeppritz AVO inversion!
Zoeppritz equation in matrix form, [Zoeppritz, 1919]

\[
\begin{bmatrix}
R_{PP}(\theta_1) \\
R_{PS}(\theta_1) \\
T_{PP}(\theta_1) \\
T_{PS}(\theta_1)
\end{bmatrix} = 
\begin{bmatrix}
-\sin \theta_1 & -\cos \phi_1 & \sin \theta_2 & \cos \phi_2 \\
\cos \theta_1 & -\sin \phi_1 & \cos \theta_2 & -\sin \phi_2 \\
\sin 2\theta_1 & \frac{V_{P1}}{V_{S1}} \cos 2\phi_1 & \frac{\rho_2 V_{S2}^2 V_{P1}}{\rho_1 V_{S1}^2 V_{P2}} \cos 2\phi_2 & \frac{\rho_2 V_{S2} V_{P1}}{\rho_1 V_{S1} V_{P2}} \cos 2\phi_2 \\
-\cos 2\phi_1 & \frac{V_{S1}}{V_{P1}} \sin 2\phi_1 & \frac{\rho_2 V_{S2} V_{P1}}{\rho_1 V_{S1} V_{P2}} \sin 2\phi_2 & -\frac{\rho_2 V_{S2}}{\rho_1 V_{P1}} \sin 2\phi_2
\end{bmatrix}^{-1} 
\begin{bmatrix}
\sin \theta_1 \\
\cos \theta_1 \\
\sin 2\theta_1 \\
\cos 2\phi_1
\end{bmatrix}
\]

- Six parameters: $V_{P1}$, $V_{P2}$, $V_{S1}$, $V_{S2}$, $\rho_1$, and $\rho_2$
- Four angles: $\theta_1$, $\theta_2$, $\phi_1$, and $\phi_2$
- Non-linear equations
- Reformulation of the Zoeppritz equation & Adjoint state technique
Reformulation of the full Zoeppritz equation [Lavaud et al., 1999]

[Lavaud et al., 1999] rewrote the full Zoeppritz equation in terms of following:

- Three contrast parameters: $e_p$, $e_s$, and $e_d$
- Average values: $\chi$

The expressions are of the form:

$$
\begin{align*}
e_p &= (\alpha_2^2 - \alpha_1^2)/(\alpha_2^2 + \alpha_1^2) \\
e_s &= (\beta_2^2 - \beta_1^2)/(\beta_2^2 + \beta_1^2) \\
e_d &= (\rho_2 - \rho_1)/(\rho_2 + \rho_1) \\
\chi &= 2(\bar{\beta}^2)/(\bar{\alpha}^2)
\end{align*}
$$

Effective implementation by reducing six ($\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$, $\rho_1$ and $\rho_2$) to four parameters ($e_p$, $e_s$, $e_d$, and $\chi$).
Exact PP-reflection coefficient:

\[ R_{PP} = \frac{P - Q}{P + Q} \]  

(3)

where variables P, Q, and other variables are functions of \( e_p, e_s, e_d \), and \( \chi \) described in Table 1.
Reformulation of the full Zoeppritz equation [Lavaud et al., 1999]

**Table 1:** Intermediate variables for equation 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$e_s + e_d$</td>
</tr>
<tr>
<td>$f$</td>
<td>$1 - e_d^2$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>$\chi(1 + e_p)$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$\chi(1 - e_p)$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$\frac{2}{1-e_s}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$\frac{2}{1+e_s}$</td>
</tr>
<tr>
<td>$q^2$</td>
<td>$S_1 \sin^2 \theta$</td>
</tr>
<tr>
<td>$D$</td>
<td>$eq^2$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$\sqrt{S_1 - q^2}$</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$\sqrt{S_2 - q^2}$</td>
</tr>
<tr>
<td>$N_1$</td>
<td>$\sqrt{T_1 - q^2}$</td>
</tr>
<tr>
<td>$N_2$</td>
<td>$\sqrt{T_2 - q^2}$</td>
</tr>
<tr>
<td>$A$</td>
<td>$e_d - D$</td>
</tr>
<tr>
<td>$B$</td>
<td>$1 - K$</td>
</tr>
<tr>
<td>$K$</td>
<td>$D - A$</td>
</tr>
<tr>
<td>$C$</td>
<td>$1 + K$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$M_2(C^2N_2 + fN_1) + 4q^2A^2$</td>
</tr>
<tr>
<td>$P$</td>
<td>$M_1(B^2N_1 + fN_2) + 4eDM_1M_2N_1N_2$</td>
</tr>
</tbody>
</table>
Inverse problem as the minimization of a residual error function $E$

$$E(x) = \frac{1}{2} \sum_{i=1}^{N_{obs}} \| R^c_{i PP}(x) - R^m_{i PP} \|^2,$$

where

- $R^m_{i PP}$: Observed (measured) $R_{PP}$ at $\theta_i$
- $R^c_{i PP}$: Forward-modeled (computed) $R_{PP}$ at $\theta_i$
- $N_{obs}$: Number of observations ($\theta_i$)
- $x$: Set of model parameters, $e_p$, $e_s$, $e_d$, and $\chi$ ($\therefore x \in \mathbb{R}^4$)

- Minimization of $E(x)$
- Computations of $\nabla_x E$: $\nabla_x R_{PP}$ [Lavaud et al., 1999, Lim et al., 2017]
- By applying adjoint state technique [Burger and Chavent, 1979]
Case Study: Avalon Shale
Upper Layer: Isotropic layer (known \( V_P, V_S, \) and \( \rho \))
Lower Layer: Target VTI shale layer (unknown \( V_P, V_S, \) and \( \rho \))

- Model based on Avalon shale from a vertical well in the Delaware basin.
- Limestone (Upper Layer) / Avalon shale (Lower Layer).
- Kerogen (v/v): 0, 0.1, 0.2, and 0.3.
- AVO data range: \( 0^\circ \) - \( 20^\circ \), and \( 0^\circ \) - \( 60^\circ \).
Well A: $V_p\,(90^\circ)$, $V_s\,(90^\circ)$, $V_p\,(0^\circ)$, $V_s\,(0^\circ)$, $\rho$, TOC for 122 data points.

Provided by Chevron
Data: $V_P$, $V_S$, $\rho$, and Anisotropic AVO response w.r.t. organic richness

$V_P (90^\circ)$ & $V_P (0^\circ)$

$V_S (90^\circ)$ & $V_S (0^\circ)$

$\rho = -1.17 \cdot \text{Kerogen} + 2.54$

Anisotropic AVO response
Inversion result: $V_P$, $V_S$, and $\rho$

**VP**, **VS**, and $\rho$

**AVO input (0° ~ 20°)**

**VP**

**VS**

**$\rho$**

**AVO input (0° ~ 60°)**

**VP**

**VS**

**$\rho$**

---

Inversion result: $V_P$, $V_S$, and $\rho$
Summary of inversion results & Workflow to estimate seismic anisotropy

Zoeppritz AVO Inversion Results
(Near Angle Range)

Vertical Vp & Vs
Correct Density

Determine Anisotropy ($\varepsilon$ & $\gamma$)

Horizontal Vp & Vs
Underestimated Density

Zoeppritz AVO Inversion Results
(Far Angle Range)

Correction by $\varepsilon$
Estimation of seismic anisotropy ($\varepsilon$ and $\gamma$)

Thomsen parameters ($\varepsilon$ and $\gamma$) [Thomsen, 1986]:

$$\varepsilon = \frac{c_{11} - c_{33}}{2c_{33}} = \frac{V_P^2(90^\circ) - V_P^2(0^\circ)}{2V_P^2(0^\circ)}$$

$$\gamma = \frac{c_{66} - c_{44}}{2c_{44}} = \frac{V_{S\perp}^2(90^\circ) - V_S^2(0^\circ)}{2V_S^2(0^\circ)}$$

(5)

where $c_{ij}$ are elastic stiffness coefficient:

$$c_{11} = \rho V_P^2(90^\circ)$$

$$c_{33} = \rho V_P^2(0^\circ)$$

$$c_{44} = \rho V_S^2(0^\circ)$$

$$c_{66} = \rho V_{S\perp}^2(90^\circ)$$

$$c_{12} = c_{11} - 2c_{66}$$

$$c_{13} = -c_{44} + \sqrt{4\rho^2 V_P^4(45^\circ) - 2\rho V_P^2(45^\circ)(c_{11} + c_{33} + 2c_{44}) + (c_{11} + c_{44})(c_{33} + c_{44})}$$

(6)
Estimation of seismic anisotropy ($\epsilon$ and $\gamma$)

\[ \epsilon \]

\[ \gamma \]

Kerogen (v/v)

Model $\epsilon$

Estimated $\epsilon$ (by the Workflow)

Model $\gamma$

Estimated $\gamma$ (by the Workflow)
Estimation of geomechanical properties ($E$ and $\nu$)

Young's modulus and Poisson's ratio of VTI medium are expressed with elastic stiffness coefficients [King, 1964, Banik et al., 2012] as following:

\[
E_V = \frac{c_{33}(c_{11} - c_{66}) - c_{13}^2}{c_{11} - c_{66}} \quad (= E_3)
\]

\[
E_H = \frac{4c_{66}(c_{33}(c_{11} - c_{66}) - c_{13}^2)}{c_{11}c_{33} - c_{13}^2} \quad (= E_1 = E_2)
\]

\[
\nu_V = \frac{c_{13}}{2(c_{11} - c_{66})} \quad (= \nu_{31} = \nu_{32})
\]

\[
\nu_{HV} = \frac{2c_{13}c_{66}}{c_{11}c_{33} - c_{13}^2} \quad (= \nu_{13} = \nu_{23})
\]

\[
\nu_{HH} = \frac{c_{33}(c_{11} - 2c_{66}) - c_{13}^2}{c_{11}c_{33} - c_{13}^2} \quad (= \nu_{12} = \nu_{21})
\]

\[
E_H = 4c_{66}(c_{33}(c_{11} - c_{66}) - c_{13}^2) \quad \frac{c_{11}c_{33} - c_{13}^2}{c_{11}c_{33} - c_{13}^2}
\]

\[
\nu_{HV} = \frac{2c_{13}c_{66}}{c_{11}c_{33} - c_{13}^2} \quad (= \nu_{13} = \nu_{23})
\]

\[
\nu_{HH} = \frac{c_{33}(c_{11} - 2c_{66}) - c_{13}^2}{c_{11}c_{33} - c_{13}^2} \quad (= \nu_{12} = \nu_{21})
\]
Estimation of geomechanical properties ($E$ and $\nu$)
Estimation of TOC from density (by the workflow)

Why density for TOC estimation?

\[ \rho = -1.17 \cdot \text{Kerogen} + 2.54 \]
Conclusions
Conclusions

• Nonlinear Zoeppritz AVO inversion
  • Horizontal $V_P$, $V_S$, and underestimated $\rho$ (with far AVO angle range)
  • Vertical $V_P$, $V_S$, and correct $\rho$ (with near AVO angle range)

• Workflow to estimate
  • seismic anisotropy ($\epsilon$ and $\gamma$)
  • geomechanical properties ($E$ and $\nu$)
  • organic abundance (TOC)

• Potential Benefits & Business case of this research
  • Defining sweet spots for shale reservoirs in terms of ‘fracability’, ‘completion’ quality, and ‘reservoir’ quality
  • Optimization of well placement, and stimulated reservoir volume (SRV)
Funding and necessary data for this research were provided by Chevron through the Chevron Basin Modeling Center of Research Excellence (CBM CoRE) research program in the Berg-Hughes Center at Texas A&M University.
Theory [Lim et al., 2017]
Estimated Inversion Results

\[
R_{PP}(\theta) = A + B \sin^2 \theta + C \sin^2 \theta \tan^2 \theta. \tag{8}
\]

[Rüger, 1997] (for Weak Anisotropy) \quad [Wiggins et al., 1983] (for Isotropy)

\[
C = \frac{1}{2} \left[ \frac{\Delta V_P(0^\circ)}{V_P(0^\circ)} + \Delta \epsilon \right], \quad C = \frac{1}{2} \frac{\Delta V_P}{V_P}. \tag{9, 10}
\]

Comparison of C in equations 9 and 10 leads:

\[
V_{P2}^{EST} \approx V_{P2}^{VTI}(90^\circ) + \frac{\left[V_{P1}^{ISO} - V_{P2}^{VTI}(90^\circ)\right]^2}{4 V_{P1}^{ISO}} \varepsilon_2. \tag{11}
\]

For small $\varepsilon$ (weak anisotropy):

\[
V_{P2}^{EST} \approx V_{P2}^{VTI}(90^\circ). \tag{12}
\]
Estimated Inversion Results

\[ R_{PP}(\theta) = A + B \sin^2 \theta + C \sin^2 \theta \tan^2 \theta. \]

[Rüger, 1997] (for Weak Anisotropy) \[ A = \frac{1}{2} \frac{\Delta Z(0^\circ)}{\bar{Z}(0^\circ)} \] (13) \[ A = \frac{1}{2} \frac{\Delta Z}{\bar{Z}} = \frac{1}{2} \frac{\Delta (\rho V_P)}{\bar{\rho} V_P} \] (14)

[Wiggins et al., 1983] (for Isotropy)

Comparison of \( A \) in equations 13 and 14, and setting \( V_{P2}^{EST} = V_{P2}^{VTI}(90^\circ) \) in equation 12 leads:

\[ \rho_2^{EST} \approx \frac{V_{P2}^{VTI}(0^\circ)}{V_{P2}^{VTI}(90^\circ)} \rho_2^{VTI} = \frac{1}{1 + \varepsilon_2} \rho_2^{VTI}. \] (15)
Supportive Supplementary Slides
$V_P, V_S, \text{ and } \rho \text{ from } e_p, e_s, \text{ and } e_d$

\begin{align}
V_{P2} &= \sqrt{\frac{1 + e_p}{1 - e_p}} \cdot V_{P1} \\
V_{S2} &= \sqrt{\frac{1 + e_s}{1 - e_s}} \cdot V_{S1} \\
\rho_2 &= \frac{1 + e_d}{1 - e_d} \cdot \rho_1
\end{align} \quad (16)
Sensitivity test for $V_p$ with the model of Avalon shale

$0^\circ \sim 20^\circ$

$0^\circ \sim 23^\circ$

$0^\circ \sim 26^\circ$

$0^\circ \sim 29^\circ$

$0^\circ \sim 32^\circ$

$0^\circ \sim 35^\circ$
Sensitivity analysis to understand behavior of inversion results

Kerogen: 0 (v/v), Fixed $\rho$: 2.54 (g/cc)

- $V_p(0^\circ) \& V_{SH}(0^\circ)$
- $V_p(30^\circ) \& V_{SH}(30^\circ)$
- $V_p(60^\circ) \& V_{SH}(60^\circ)$
- $V_p(90^\circ) \& V_{SH}(90^\circ)$
- $R_{PP}$ (Gibson et al., 1991)

Angle of Incidence, $\theta$ ($^\circ$)
Table 2: Two-layer models for the testing AVO inversions

<table>
<thead>
<tr>
<th>Model</th>
<th>Kerogen (v/v)</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$V_P(0^\circ)$ (km/s)</th>
<th>$V_P(90^\circ)$ (km/s)</th>
<th>$V_S(0^\circ)$ (km/s)</th>
<th>$V_{SH}(90^\circ)$ (km/s)</th>
<th>$\varepsilon$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Layer</td>
<td>0</td>
<td>2.63</td>
<td>5.05</td>
<td>5.05</td>
<td>2.90</td>
<td>2.90</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lower Layer</td>
<td>0.0</td>
<td>2.54</td>
<td>4.23</td>
<td>4.70</td>
<td>2.71</td>
<td>2.92</td>
<td>0.12</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>2.42</td>
<td>3.96</td>
<td>4.54</td>
<td>2.50</td>
<td>2.80</td>
<td>0.16</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>2.31</td>
<td>3.69</td>
<td>4.38</td>
<td>2.29</td>
<td>2.68</td>
<td>0.20</td>
<td>0.18</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>2.19</td>
<td>3.42</td>
<td>4.22</td>
<td>2.09</td>
<td>2.56</td>
<td>0.26</td>
<td>0.25</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Improvement by Joint Zoeppritz inversion [Lim et al., 2018]

\[
E(m) = \frac{1}{2} \sum_{i=1}^{N} \left( \| R_{i}^{d}_{PP} - R_{i}^{c}_{PP}(m) \|^2 + \| R_{i}^{d}_{PS} - R_{i}^{c}_{PS}(m) \|^2 \right),
\]

(17)

where

- \( R_{i}^{d} \): Observed \( R_{PP} \) & \( R_{PS} \) at \( \theta_{i} \)
- \( R_{i}^{c} \): Forward-modeled \( R_{PP} \) & \( R_{PS} \) at \( \theta_{i} \)
- \( N \): Number of \( \theta_{i} \)
- \( m \): Set of model parameters, \( e_{p}, e_{s}, e_{d} \), and \( \chi \).

- Minimization of \( E(m) \)
- Computations of \( \nabla E(m) \): \( \nabla R_{PP} \) [Lavaud et al., 1999], \( \nabla R_{PS} \) [Lim et al., 2018]
- By applying adjoint state technique [Burger and Chavent, 1979]