Antialiasing 5D Interpolation*

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Abstract

Fourier-based minimum weighted norm interpolation (MWNI) has been widely used to regularize land seismic data. It is relatively computationally fast, and easily extends to higher dimensions. However, it has difficulty to interpolate regularly missing data that are spatially aliased. Minimizing the aliasing artifacts is still a technical challenge in MWNI. I propose a new and novel method to address the aliasing problem in MWNI. The first step involves construction of a fully regular initial model in time domain, but this initial model does not require being an accurate representation of the true data model. The spectral weights derived from this regular initial model are typically not aliased and can be used to constrain the least-squares inversion in MWNI, frequency by frequency, to overcome the aliasing artifacts. This new interpolation scheme expands the capability of the conventional MWNI to handle aliased data that often associate with steeply dipping structures. Applications of this new method on 2D synthetic and 5D field data demonstrate its effectiveness to interpolate multi-dimensional data.

Introduction

Acquisition limitations such as field obstacles and economic costs often result in irregular and incomplete seismic field data. However, many common signal processing and imaging algorithms require regularly sampled data to work properly. Multi-dimensional interpolation is typically employed to generate regular data to meet the processing requirements. There are a number of available methods to perform multi-dimensional interpolation. Fourier-based algorithm is one of the most versatile approaches. It is relatively computationally fast, and easily extends to higher dimensions. Minimum weighted norm interpolation (MWNI) belongs to the Fourier-based method (Liu and Sacchi, 2004). Examples from MWNI (Trad, 2009) showed that MWNI was successful to interpolate sparse data, improve AVO analysis, and reduce migration artifacts.

Although the conventional MWNI is effective to interpolate unaliased data, it has difficulty to handle regularly missing data that are spatially aliased (Naghizadeh and Sacchi, 2010). A common strategy to minimize the aliasing artifacts in MWNI uses a low-frequency solution, which assumes to be unaliased to constrain a high-frequency solution (Hermann et al., 2000). Cary (2011) pointed out that this approach did not

provide enough constraints to overcome the aliasing problem at high-frequency solutions. In addition, if the data were aliased in one spatial dimension, the multi-dimensional interpolation did not eliminate the aliasing issue. Naghizadeh and Sacchi (2007) proposed a multistep autoregressive approach in an attempt to address the aliasing problem. However, their technique had difficulty to work properly when the data had large and highly irregular data gaps. Overcoming the aliasing artifacts in MWNI is still an outstanding challenge.

This paper proposes a new and novel method to overcome the aliasing artifacts in MWNI. The new approach consists of two key steps: the first step constructs an approximate initial model that is regular without data gaps (Chiu and Anno, 2012). The second step uses this regular initial model to compute spectral weights that constrain the least-squares solutions in MWNI. The multi-dimensional interpolation using the MWNI algorithm constrained by this regular initial model is referred to "antialiasing MWNI". Applications of this method on a 2D synthetic data and a 5D field data illustrate the effectiveness of the new interpolation scheme.

Theory

The complete unknown data "$\mathbf{x}$" is related to available incomplete data "$\mathbf{d}$" by a sampling matrix $\mathbf{T}$ as:

$$\mathbf{Tx} = \mathbf{d},$$

The complete unknown data $\mathbf{x}$, frequency by frequency, can be reconstructed by minimizing the following cost function $\mathbf{J}$, where:

$$\mathbf{J} = \| \mathbf{Tx} - \mathbf{d} \|^2 + \mu \, \mathbf{x}^H \mathbf{F}^{-1} |\mathbf{P_k}|^{-2} \mathbf{F} \, \mathbf{x},$$

and the least-squares solution with a minimum weighted norm is:

$$\mathbf{x'} = (\mathbf{T}^H \mathbf{T} + \mu \, \mathbf{F}^{-1} |\mathbf{P_k}|^{-2} \mathbf{F})^{-1} \mathbf{T}^H \mathbf{d},$$

where:

μ is a weighing factor controlling the tradeoff between the model norm and misfit of observations,
$\mathbf{H}$ is a conjugate transpose operator,
$\mathbf{x}^H$ is a conjugate transpose of $\mathbf{x}$,
$\mathbf{F}$ is a multi-dimensional forward Fourier transform,
$\mathbf{F}^{-1}$ is a multi-dimensional inverse Fourier transform, and
$\mathbf{x'}$ is a least-squares solution with a minimum weighted norm (MWNI) to recover missing data.

$|\mathbf{Pk}|^{-2}$ represents the spectral weights in frequency and wavenumber domains on the fully sampled multi-dimensional unknown seismic data $\mathbf{x}$. Since the available data $\mathbf{d}$ have missing seismic data, spectral weights cannot be computed and are typically unknown. It is crucial to compute a fully regular initial model that can be used to estimate unknown spectral weights. In general, a time-domain 2D linear interpolation along

dominant dips in each spatial dimension is adequate to construct a regular initial multi-dimensional model. The spectral weights estimated from this regular initial model are typically not aliased, and the use of the unaliased spectral weights to constrain the least-squares solution, frequency by frequency, is the key to overcome the aliasing in MWNI.

Synthetic Example

A 2D synthetic example consists of three linear events (Figure 1a). One of the dipping events becomes spatially aliased after decimating every second and third traces as well as removing a number of traces to create large data gaps (Figure 1b). The conventional MWNI using a low-frequency solution to constrain a high-frequency solution interpolated unaliased events properly, but failed to interpolate the aliased event (Figure 1c). The difference plot (Figure 1d) clearly illustrated that conventional MWNI had difficulty to handle spatially aliased data. As an illustrated example, it is intentional to construct a regular initial model that deviates considerably from the true model, showing that a good initial model is not a requirement (Figure 2a). The antialiasing MWNI interpolated both aliased and non-aliased events correctly (Figure 2b). The reconstructed data from the antialiasing MWNI almost matched the original data (Figure 2c).

F-K domain is another way to examine the aliasing artifacts. The F-K spectrum before data decimation showed three distinctive events corresponding to three events in time domain (Figure 3a). However, the data decimation caused one of the linear events to be spatially aliased as indicated by the repeated F-K spectra (Figure 3b). The conventional MWNI mapped the single aliased event into three distinctive segments instead of a single segment (Figure 3c). On the contrary, the antialiasing MWNI recovered both the aliased and unaliased events accurately (Figure 3d). The synthetic example demonstrates that antialiasing MWNI handles spatially aliased data correctly.

5D Interpolation of a Field Example

The interpolation framework of this data example is offset-vector-tile (OVT) inline, OVT crossline, inline within an OVT, crossline within an OVT, and time. Since sampling in time is always regular, the interpolation only operates on four spatial dimensions. The objective is to fill in missing data of all OVTs. Figures 4a and 4b show an example of an OVT fold map before and after the 5D antialiasing MWNI. The 5D interpolation was effective to reconstruct almost all data gaps. However, a few small data gaps were still present after 5D interpolation (Figure 4b) because we limited the aperture of the interpolation operator. In theory, it is straightforward to use a larger operator to interpolate all data gaps; but in practice, it is more prudent to limit the aperture of the interpolation operator in order to preserve the integrity of the reconstructed data. The 5D interpolation did an excellent job to reconstruct the missing data (Figures 5a and 5b). Seismic events were continuous across data gaps and followed closely the structures of the original data. Another benefit of the 5D interpolation increases the signal-to-noise ratio of the interpolated data due to random-noise reduction during the interpolation process. After stacking each migrated OVT from the original data and data after the 5D antialiasing MWNI, it is obvious that 5D interpolation significantly improves the resolution of imaging the complex structures (Figures 6a and 6b).

Conclusions

The antialiasing MWNI addresses the aliasing limitation in the conventional MWNI. It improves the fidelity of the multi-dimensional interpolation. Synthetic data testing clearly demonstrated that antialiasing MWNI handled aliased data properly while conventional MWNI failed. Its application on a 5D data set was successful to regularize a non-uniform land survey, increase data fold, and improve structural imaging. This new interpolation scheme offers new possibilities to expand the conventional MWNI capability to interpolate aliased data that typically correspond to steeply dipping structures.

Acknowledgements

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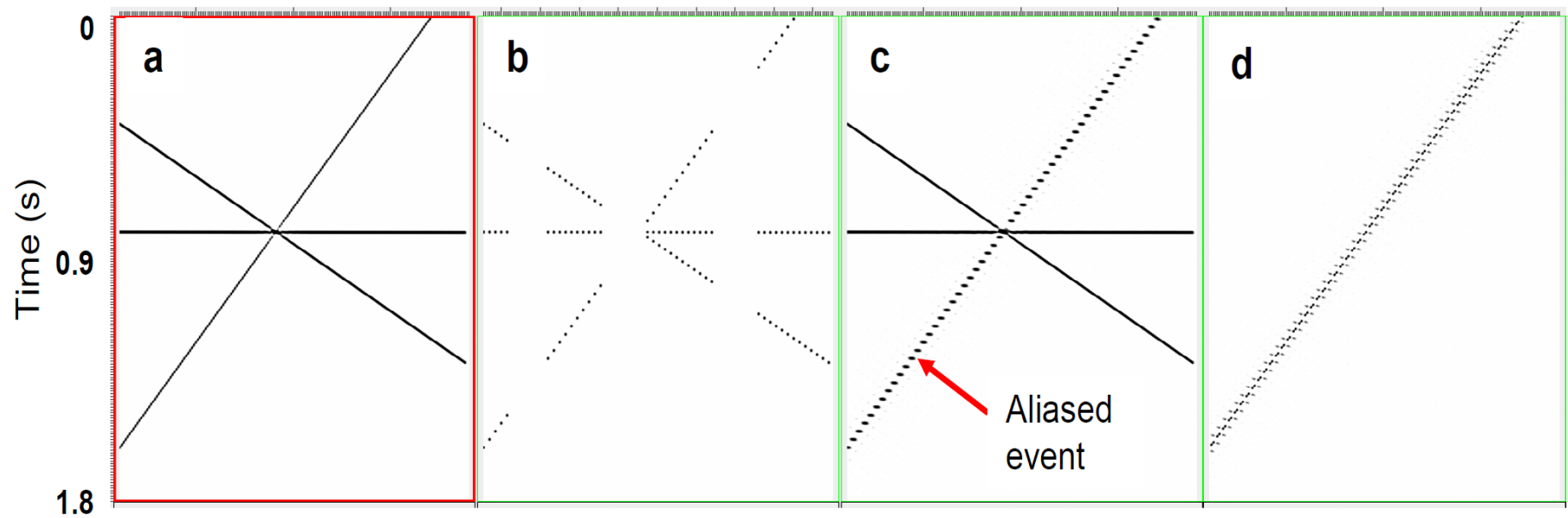
Figure 1. A 2D synthetic example in evaluating conventional MWNI performance: (a) original data consisting of three linear events, (b) decimating every second and third traces as well as removing a number of traces to create large data gaps, (c) conventional MWNI result, showing the aliasing artifacts, (d) the difference between (a) and (c).
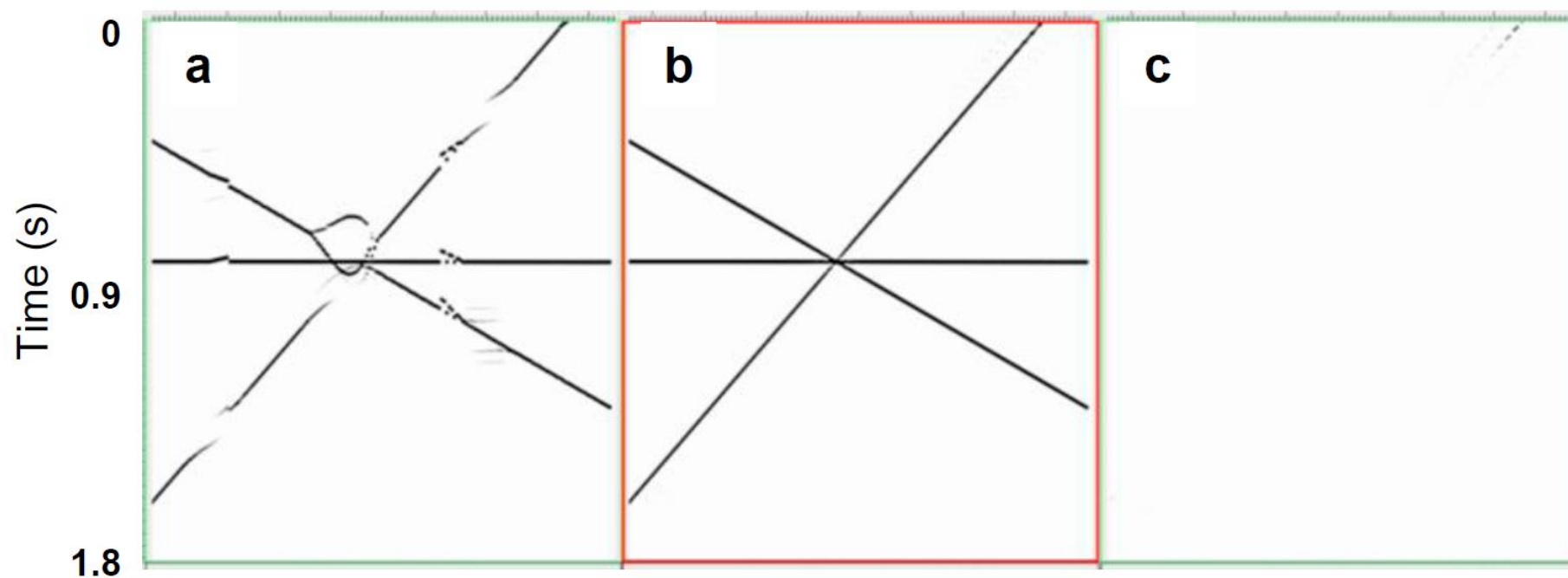
Figure 2. A 2D synthetic example in evaluating antialiasing MWNI performance: (a) a fully regular initial data model used to derive spectral weights in MWNI. It is intentional to construct an initial model that deviates considerably from the true model as a demonstration of this method, (b) antialiasing MWNI result, (c) the difference between Figure 1a and Figure 2b.
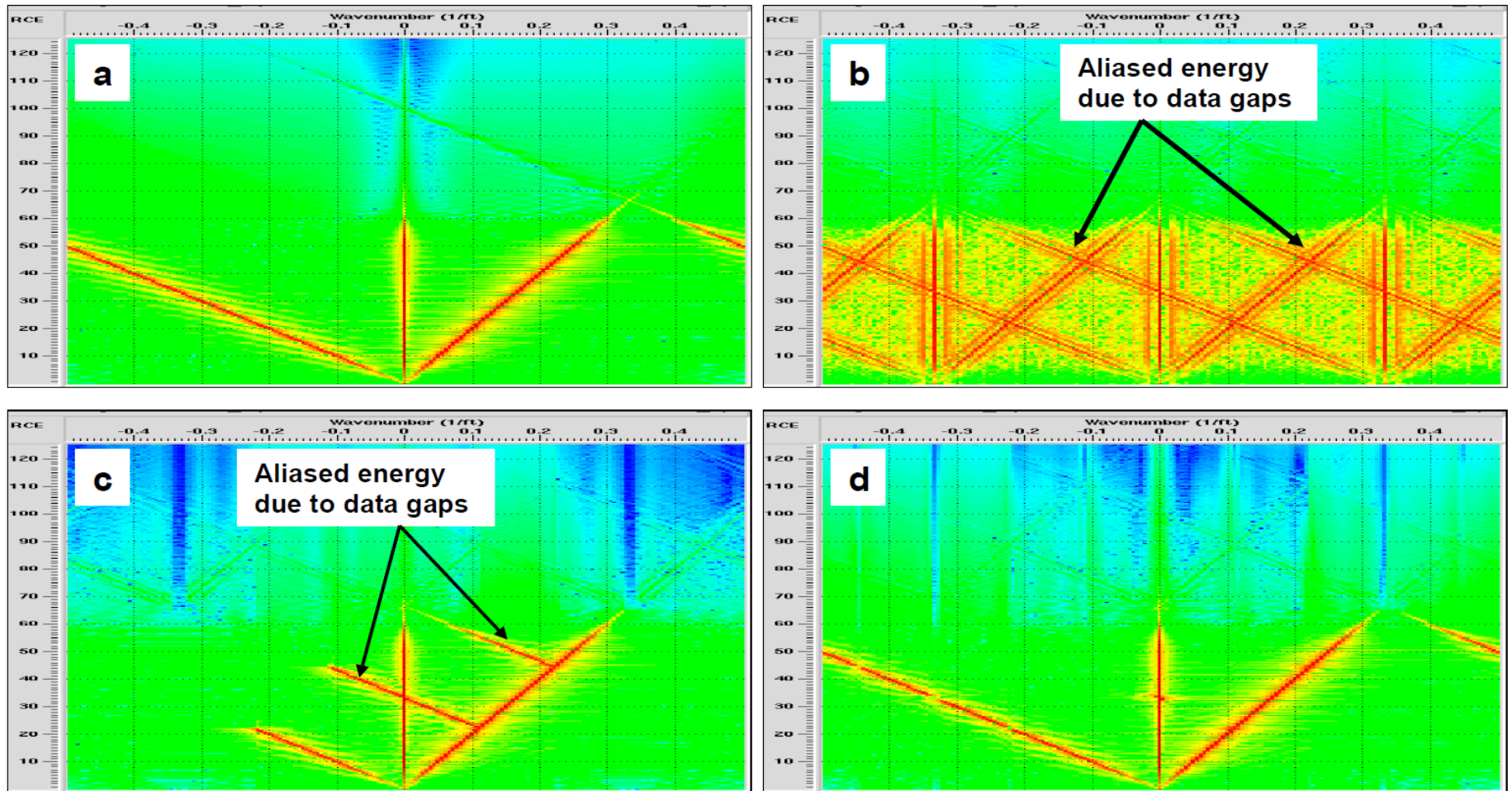
Figure 3. F-K spectrum comparisons: (a) original data without data decimation, (b) decimated data, (c) data after conventional MWNI, (d) data after antialiasing MWNI.
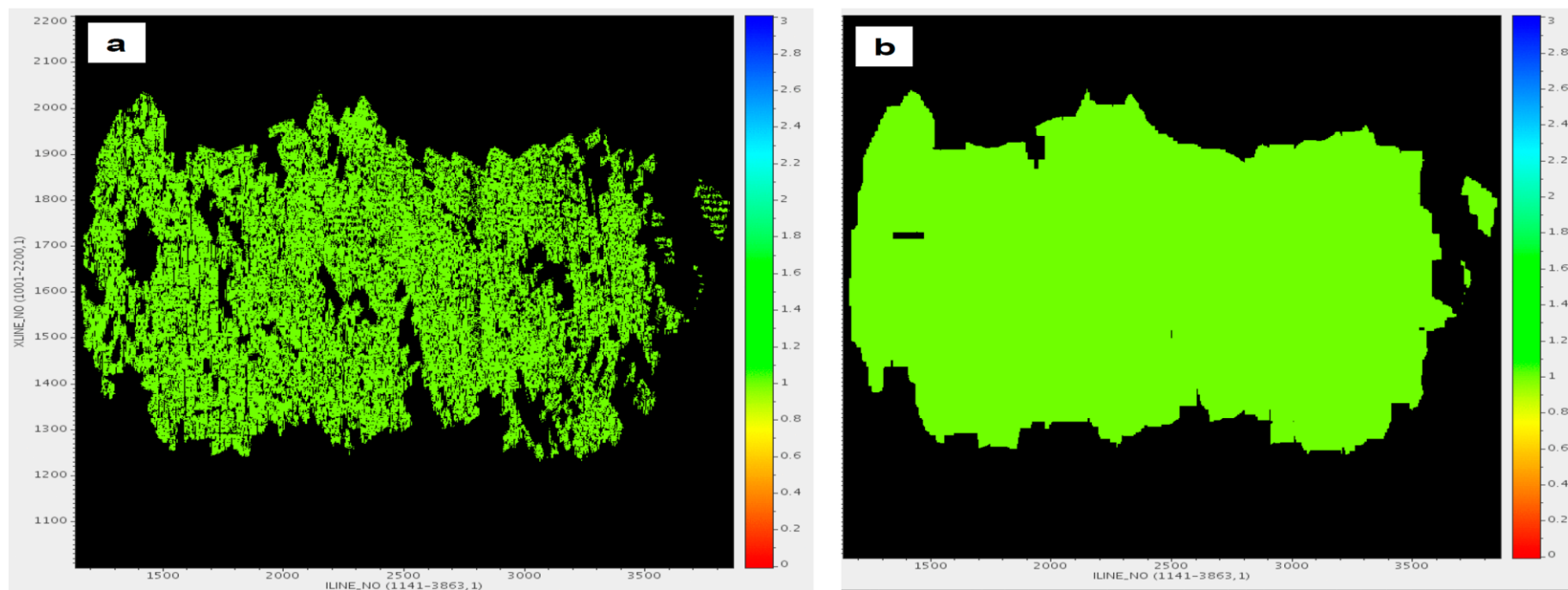
Figure 4. A fold-map example of an OVT before and after the 5D antialiasing MWNI: (a) original data, (b) data after 5D interpolation. Black-background color represents missing data.
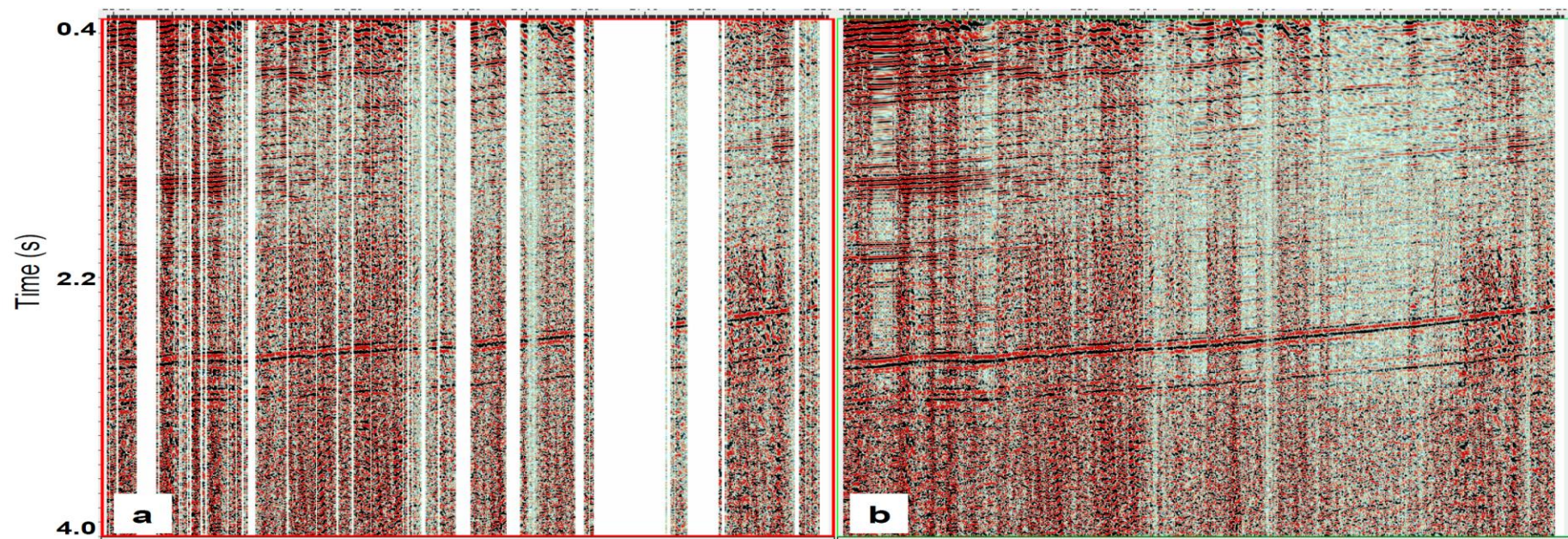
Figure 5. A typical inline profile of an OVT before and after 5D interpolation: (a) original data, (b) data after 5D interpolation.
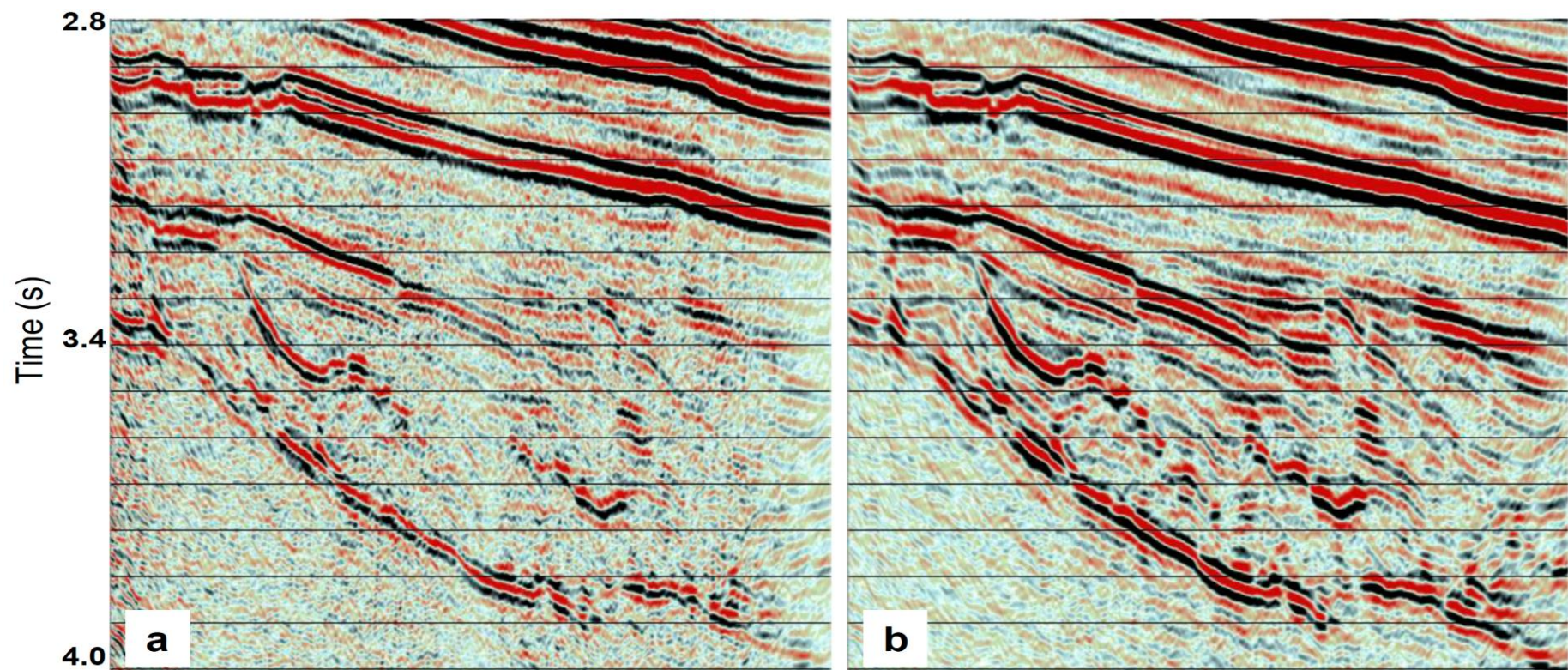
Figure 6. A typical time-migrated inline stack before and after 5D interpolation, showing a portion of complex structures: (a) original data, (b) data after 5D interpolation.