Wavelet Estimation by Non-Linear Optimization of All-Pass Operators

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Abstract

Parameterization of a mixed-phase wavelet in terms of a minimum phase wavelet and an all-pass wavelet can be used to enhance the bandwidth of the data. Deconvolution of the data with the estimated minimum phase wavelet serves this purpose. Increase in the bandwidth is a favorable effect since the cumulant matching technique, which subsequently follows to estimate the all-pass wavelet works in a more favorable regime of bandwidth to central frequency ratio. In addition, compared to the existing cumulant matching technique of wavelet estimation, the proposed technique needs to be optimized only for the all-pass wavelet with the constraint that the all-pass wavelet has a minimum phase denominator. Such a constraint makes the optimization algorithm less expensive by suitably restricting the search space.

A comparison is made between the estimation obtained from pre-whitened data (the estimated minimum phase wavelet removed from the data) and that obtained from the non-whitened data. The former case showed higher correlation measure between the estimated and the true synthetic wavelet. The algorithm is further applied to a shot gather from Mississippi canyon seismic data. The data are deconvolved with the estimated mixed-phase wavelet. A testing of the algorithm was followed by incorporating a synthetically designed phase rotated mixed phase wavelet into the deconvolved data. The proposed algorithm could effectively estimate the synthetic wavelet.

Introduction

A mixed-phase wavelet differs from its corresponding minimum phase wavelet by its phase signature. Since, second-order statistical methods do not preserve phase signature, there is a need to look for higher order statistical methods in order to estimate the mixed phase wavelet. Cumulants, which involve higher order statistical calculations, are known to preserve the amplitude and phase signatures of the wavelet. Unlike the third order cumulants, the fourth order cumulants do not vanish when the reflectivity series is symmetric (Walden and Hosken, 1986).

Tugnait (1987) proposed a fourth-order cumulant matching technique to estimate a mixed-phase moving average wavelet. Lazear (1993) applied this technique to the real seismic data. Velis and Ulrych (1996) applied the technique with a non-linear optimization approach. They estimated the mixed-phase wavelet from the fourth-order cumulant of the trace by means of Very Fast Simulated Annealing (VFSA) optimization method. They showed the dependence of the cumulant matching technique on the ratio of bandwidth to central frequency of the data.

An improvement over the previous cumulant matching approaches to mixed phase deconvolution is proposed here. Parameterization of the mixed-phase wavelet as a convolution of a minimum phase wavelet with an all-pass wavelet can significantly simplify the problem. Deconvolving the seismic trace by the estimated minimum phase wavelet helps in broadening the bandwidth of the deconvolved data. This is a desirable effect. As pointed out by Velis and Ulrych (1996), a proper estimation of the mixed-phase wavelet by cumulant matching technique is

possible when the ratio of the bandwidth to central frequency is greater than one and preferably close to two. Hence, deconvolution by the estimated minimum-phase wavelet works favorably for the cumulant matching technique. Optimization for the all-pass wavelet is performed by means of the Metropolis algorithm (Sen and Stoffa, 1995).

Technical Overview

With the assumptions that the reflectivity series is non-Gaussian, stationary and a statistically independent random process, the fourth order cumulant of the trace is equal to, within a scale factor, the fourth order moment of the wavelet. With the same analogy, it can be said that, when the wavelet is parameterized as a convolution of a minimum phase wavelet and an all-pass wavelet, the fourth-order cumulant of the whitened trace (deconvolved by the minimum phase wavelet) is equal to, within a scale factor, the fourth-order moment of the all-pass wavelet. The minimum phase wavelet estimated from the autocorrelation of the trace has the same amplitude spectrum as that of the true mixed phase wavelet. Thus, deconvolving the trace with the minimum phase wavelet not only removes the wavelet amplitude spectrum from the data but also enhances the bandwidth. This allows the optimization algorithm to work in a more favorable regime of bandwidth to central frequency ratio. The whitened trace now contains only the phase information of the wavelet. Hence, an all pass wavelet remains to be optimized from the whitened data.

The Z-transform of the all-pass wavelet ($F(Z)$) that needs to be estimated can be written as (Porsani and Ursin, 1998)

$$F(Z) = Z^p \frac{B(Z^{-1})}{B(Z)} \qquad\qquad (1)$$

where, $B(Z) = b_0 + b_1 Z + b_2 Z^2 + ... + b_a Z^P$ and the term Z^p accounts for the time shift required to make the all-pass wavelet causal. It is important to mention here that the time series $b_t = b_0, b_1, ... , b_p$ is minimum phase. This is a very simple parameterization with $b_t = b_0, b_1, ... , b_p$ as unknowns. For this problem, the term Z^p in the equation (1) is not important because it only accounts for the time shift in the final estimator of the wavelet.

Synthetic Data Examples

The proposed algorithm for estimating the mixed-phase wavelet is tested by convolving a synthetic mixed phase wavelet with a reflectivity series having Laplacian mixture distribution. The reflectivity sequence has a length N = 250. The synthetic trace does not contain any noise component. The minimum phase wavelet is estimated from the autocorrelation of the trace by the Wiener-Levinson algorithm. The whitened trace is obtained by deconvolving the data with the estimated minimum phase wavelet. The whitened trace contains only the phase information of the wavelet as the amplitude information has been effectively removed by the deconvolution. Hence, the technique of cumulant matching reduces to the matching of the fourth-order moment of the all-pass wavelet and the fourth-order cumulant of the whitened trace.

The phase estimation by cumulant matching technique is performed by using the Metropolis algorithm, a non-linear optimization tool. The algorithm is used to optimize for the model parameters $b_t = b_0, b_1, ..., b_p$ which form the denominator term of the all-pass wavelet (equation 1). The unknowns here are the length of b_t and its coefficients. The optimization is done by fixing the length of b to 4. This is the minimum length of b that can produce a distribution of imaginary and real roots. As mentioned earlier, b_t is minimum phase. This a priori information was incorporated in the Metropolis algorithm by using the Kolmogoroff technique. The cost function for the optimization is given by

$$J = \sum_{\tau_1} \sum_{\tau_2} \sum_{\tau_3} \left[C_4^{\%s}(\tau_1, \tau_2, \tau_3) - M_4^{\%w}(\tau_1, \tau_2, \tau_3) \right]^2 \qquad (2)$$

Where, $C_4^{\%s}(\tau_1, \tau_2, \tau_3)$ is the fourth-order trace cumulant (normalized by the central lag cumulant) and $M_4^{\%w}(\tau_1, \tau_2, \tau_3)$ is the fourth-order wavelet moment (normalized by the central lag moment).

Fig. 1(a) shows the synthetic data with an underlying reflectivity sequence that has a Laplacian mixture distribution. Number of data $N = 250$. Fig. 1(b) shows the true synthetic mixed phase wavelet. Fig. 1(c) shows the data after being deconvolved by the minimum phase wavelet. Fig. 1(d) shows the estimated minimum phase wavelet. Fig. 2(a) shows the true wavelet and Fig. 2(b) shows the estimated mixed phase wavelet. The optimization was done with a model length $p = 4$. The estimated wavelet has a correlation of 0.99 with the true wavelet.

Comparison of the Results

A comparison is made between the estimation of the mixed-phase wavelet with the proposed algorithm and that directly obtained from the data. A synthetic mixed phase wavelet with a bandwidth to central frequency ratio of 0.5 is designed for the purpose of illustration here. A synthetic trace is generated by convolving the true synthetic mixed phase wavelet with a reflectivity sequence of length $N = 250$. The underlying reflectivity sequence has Laplacian mixture distribution. The synthetic trace, thus generated, is whitened by deconvolving with the estimated minimum phase wavelet. Fig. 3(a) shows the true synthetic data. Fig. 3(b) shows the true synthetic mixed phase wavelet. Fig. 3(c) shows the data after being deconvolved with the minimum phase wavelet. Fig. 3(d) shows the estimated minimum phase wavelet. Fig. 4(a) shows the true synthetic mixed phase wavelet and fig. 4(b) shows the estimated mixed phase wavelet obtained from the whitened data. A correlation of 0.99 is observed between the estimated and true wavelets. Fig. 5(a) shows the true wavelet and fig. 5(b) shows the estimated wavelet obtained from the non-whitened data. A correlation measure of 0.89 is observed between the true and the estimated wavelets. The model length p used in both the cases is equal to 4. This observation corroborates the fact that the cumulant matching technique works better when the ratio of bandwidth to central frequency of the data is greater than 1, and preferably close to 2.

Real Data Example

A shot gather from Mississippi canyon is considered for testing the algorithm on real data. Data with 21 traces and 251 time samples are windowed from a shot gather. Average cumulant is calculated for the data window and incorporated in the cost function (equation 2) for the

estimation of the all-pass wavelet from the pre-whitened data. Data are pre-whitened by deconvolving with the estimated minimum phase wavelet.

Fig. 6(a) shows the estimated minimum phase wavelet from the real data. Fig. 6(b) shows the estimated mixed phase wavelet for a model length p = 4. Fig. 7(a) shows the roots of the estimated minimum phase wavelet plotted on the Z-plane. Fig. 7(b) shows the roots of the estimated mixed phase wavelet plotted on the Z- plane. It is noticed that the mixed phase wavelet has one imaginary root (complex and its conjugate taken together) and one real root. Fig. 8(a) shows the real data. Fig. 8(b) shows the data after being deconvolved with the estimated mixed phase wavelet. Fig. 8(c) shows the data after being deconvolved by the minimum phase wavelet. It is observed that a minimal phase change that could be optimized by the algorithm resulted in a significant improvement in the deconvolved data. The deconvolved data is further convolved with a 900 phase rotated mixed phase synthetic wavelet. The algorithm is applied to this data to check if the algorithm could estimate the phase rotated wavelet (Hargreaves, 1994). Fig 9(a) shows the true phase rotated wavelet. Fig 9(b) shows the estimated minimum phase wavelet. Fig (9c) shows the recovered 900 phase rotated wavelet from the data. The model length for optimization was 4.

Conclusions

Cumulant matching technique works well when the bandwidth to central frequency ratio in the data is greater than 1. The technique is best suitable when this ratio is close to 2 i.e. the data is full band. The proposed technique separates the minimum phase part of the wavelet from the data by deconvolving the data with an estimated minimum phase wavelet. As a result, the deconvolved data contains only the phase signature of the mixed-phase wavelet. This also allows the cumulant matching technique to work in a favorable regime of the bandwidth to central frequency ratio. The synthetic examples showed that the Metropolis algorithm can be used quite effectively to estimate the all-pass wavelet and hence the mixed phase wavelet. The paper also presented a comparison when the wavelet is estimated from the whitened data and non-whitened data. The proposed algorithm allows for whitening the data using the estimated minimum phase wavelet. It is observed that when the data are severely band limited, the estimated mixed phase wavelet had relatively poor correlation with the true wavelet. However, suitable parameterization of the wavelet and subsequent whitening of the data, improved the estimation of the mixed phase wavelet. This is an encouraging result for the proposed algorithm.

Acknowledgements

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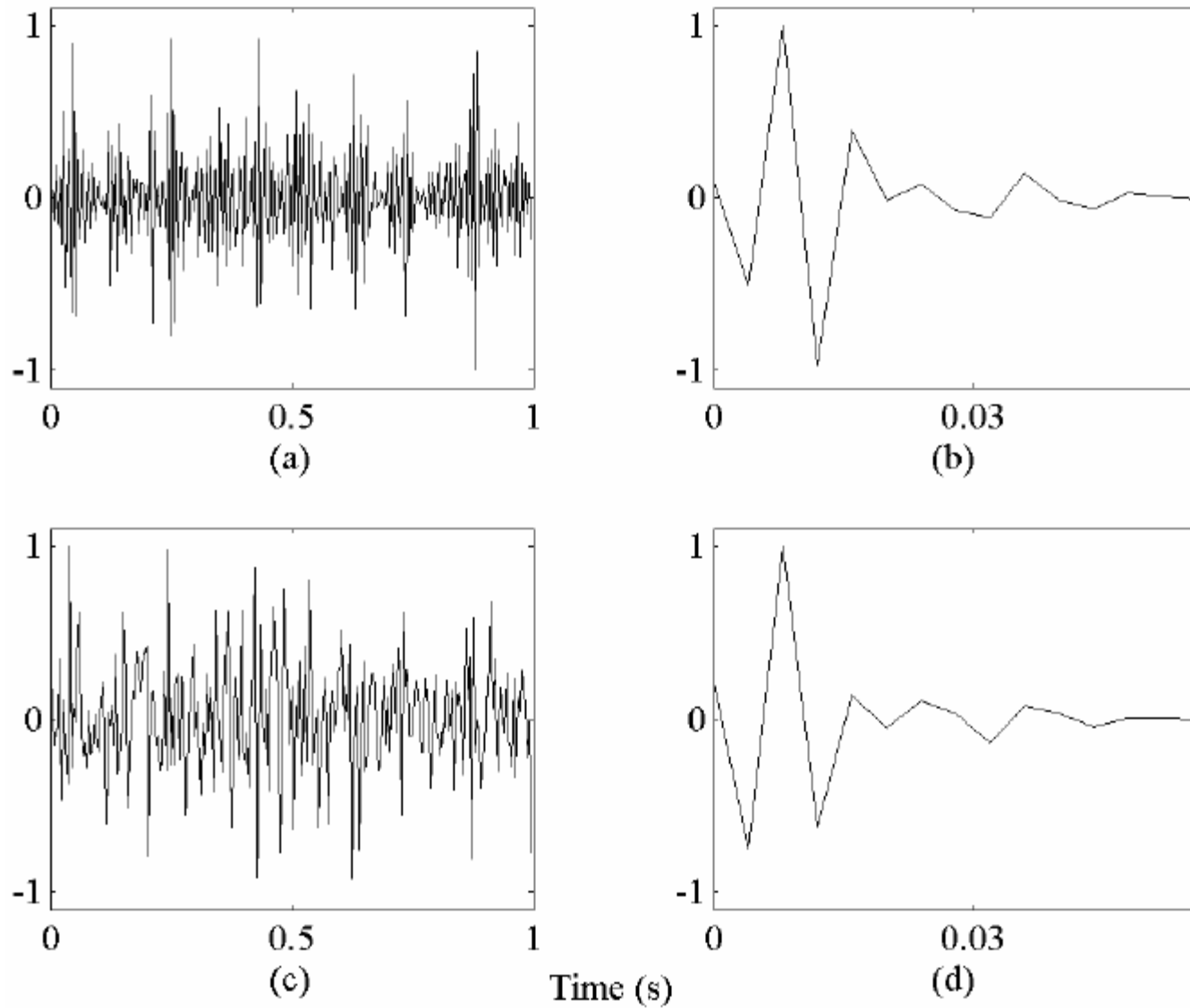
Figure 1. (a) The synthetic data. (b) The true synthetic wavelet. (c) Whitened data. (d) The estimated minimum phase wavelet. Number of data N=250.
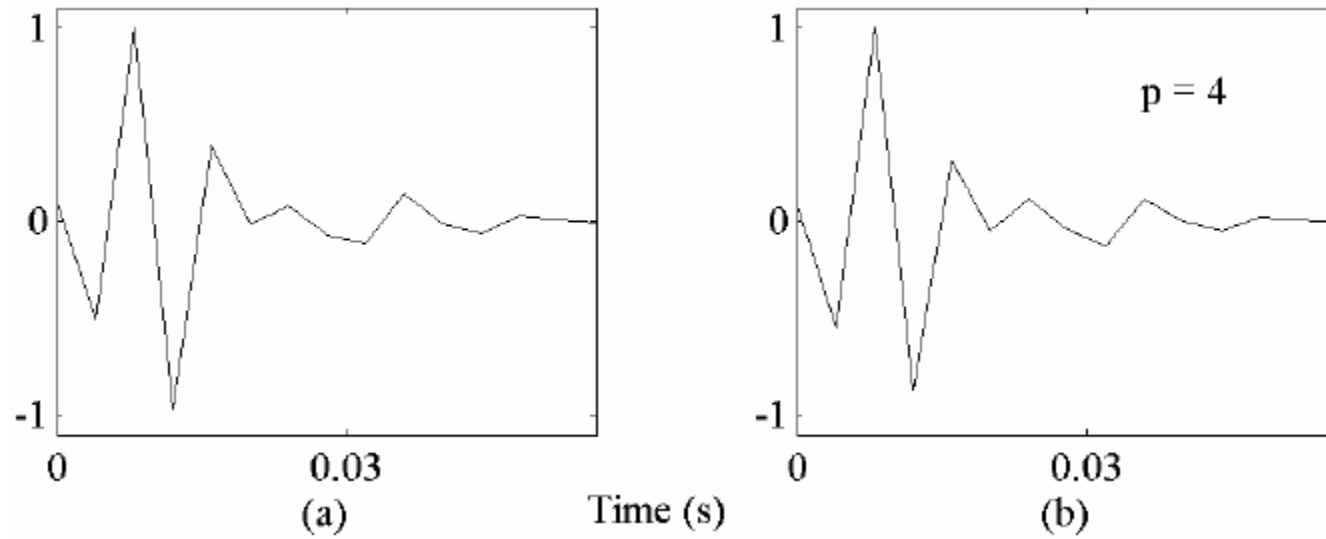
Figure 2. (a) The true mixed phase wavelet. (b) The estimated mixed phase wavelet for model length p = 4.
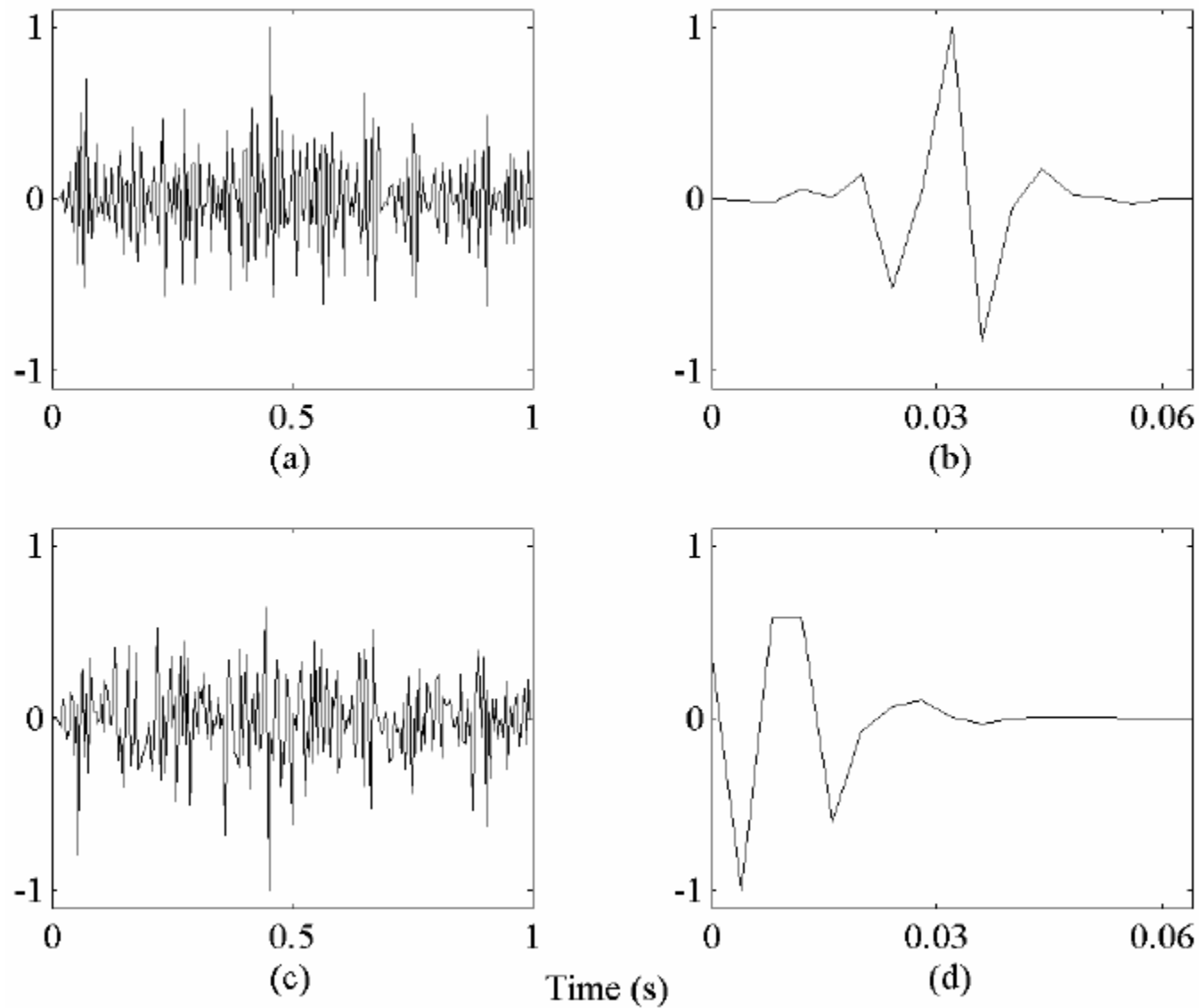
Figure 3. (a) The synthetic data. (b) The true mixed phase wavelet with a bandwidth to central frequency ratio 0.5. (c) Whitened data. (d) Estimated minimum phase wavelet.
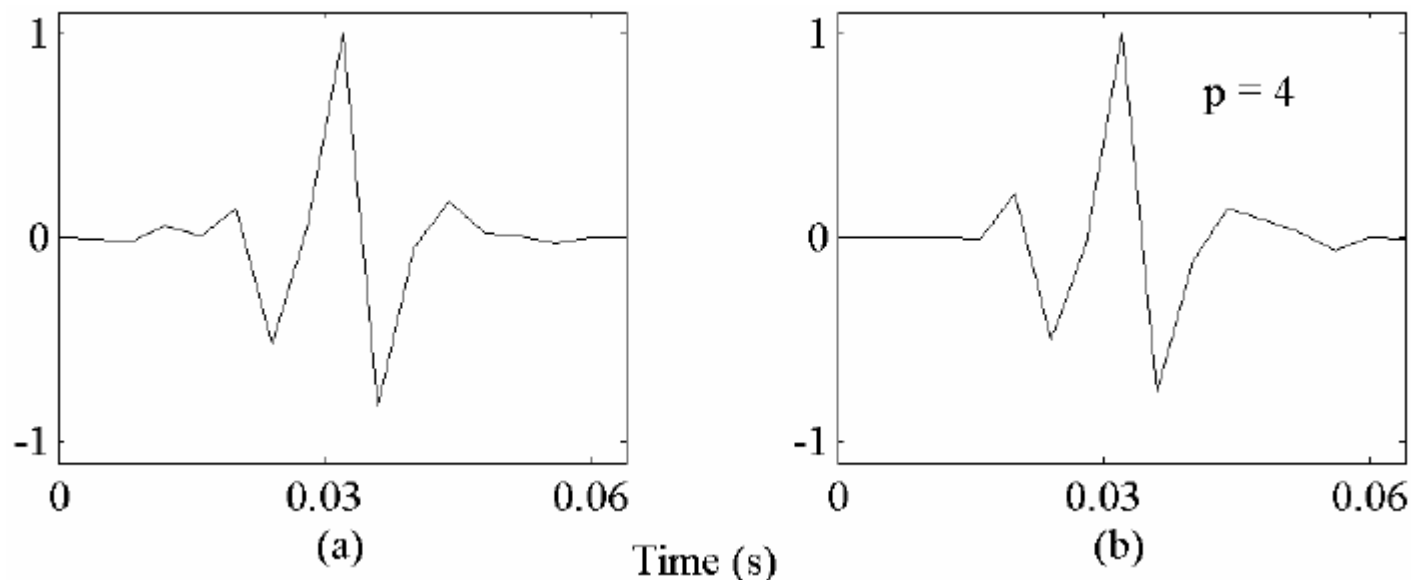
Figure 4. (a) The true mixed phase wavelet. (b) The estimated mixed phase wavelet from whitened data. The model length p = 4. Correlation = 0.99.
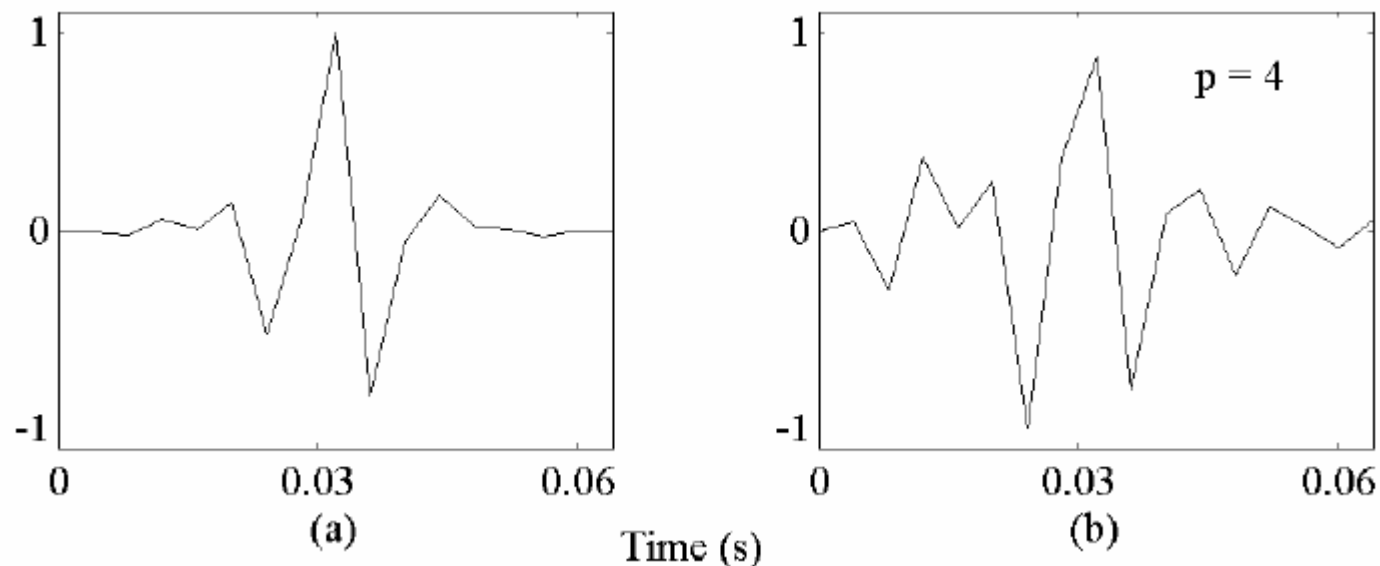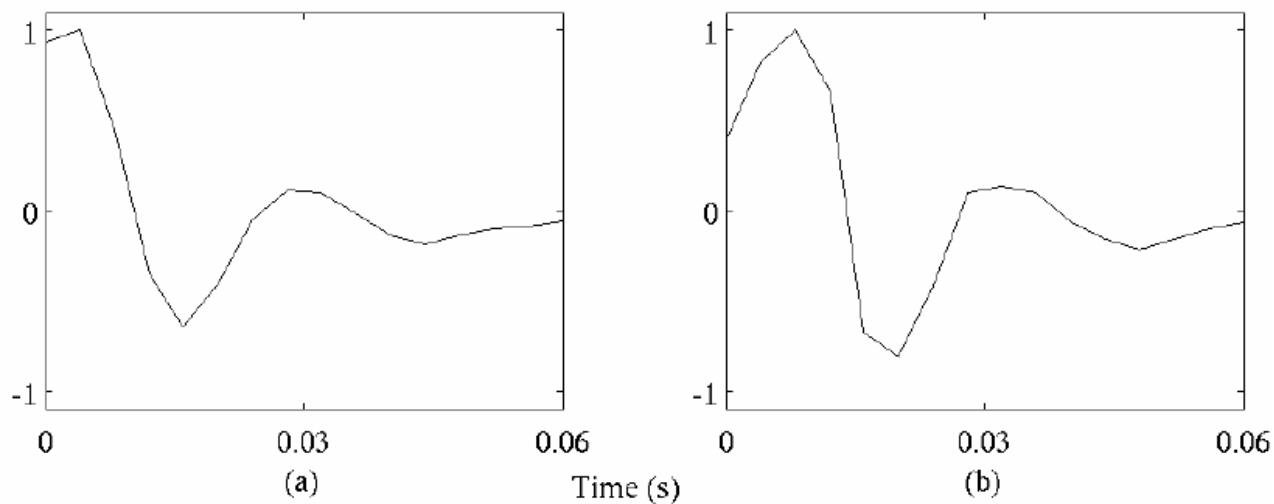

Figure 5. (a) The true mixed phase wavelet. (b) The estimated mixed phase wavelet without whitening the data. The model length p = 4. Correlation = 0.89.

Figure 6. Real data example. (a) The estimated minimum phase wavelet. (b) The estimated mixed phase wavelet. Number of data N = 250.
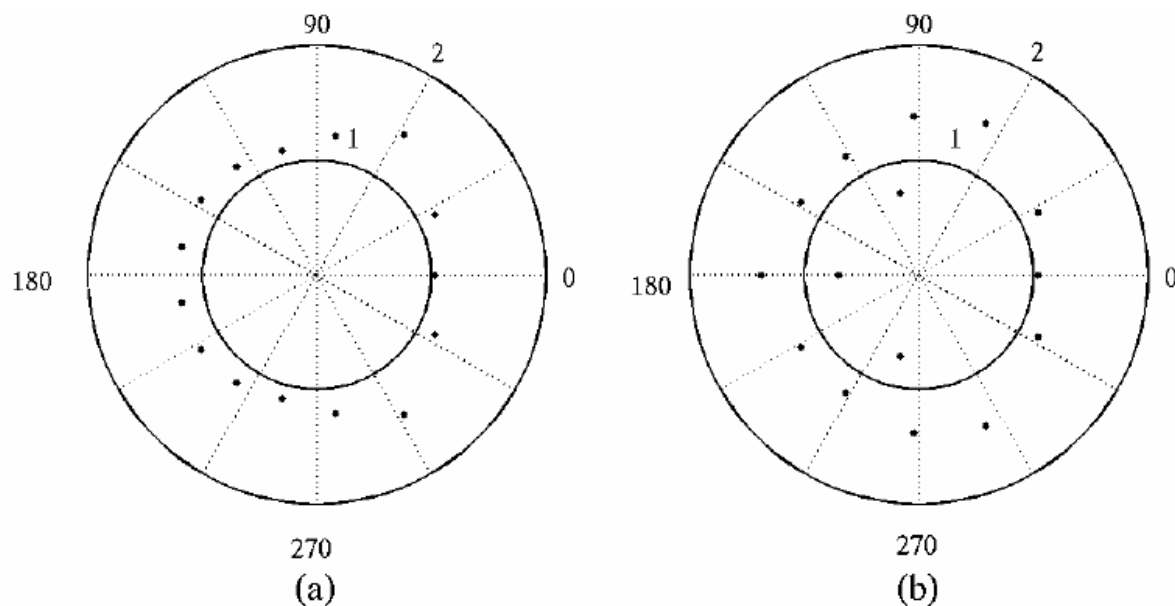

Figure 7. The roots. (a) The roots of the estimated minimum phase wavelet plotted on the Z- plane. (b) The roots of the estimated mixed phase wavelet plotted on the Z- plane.
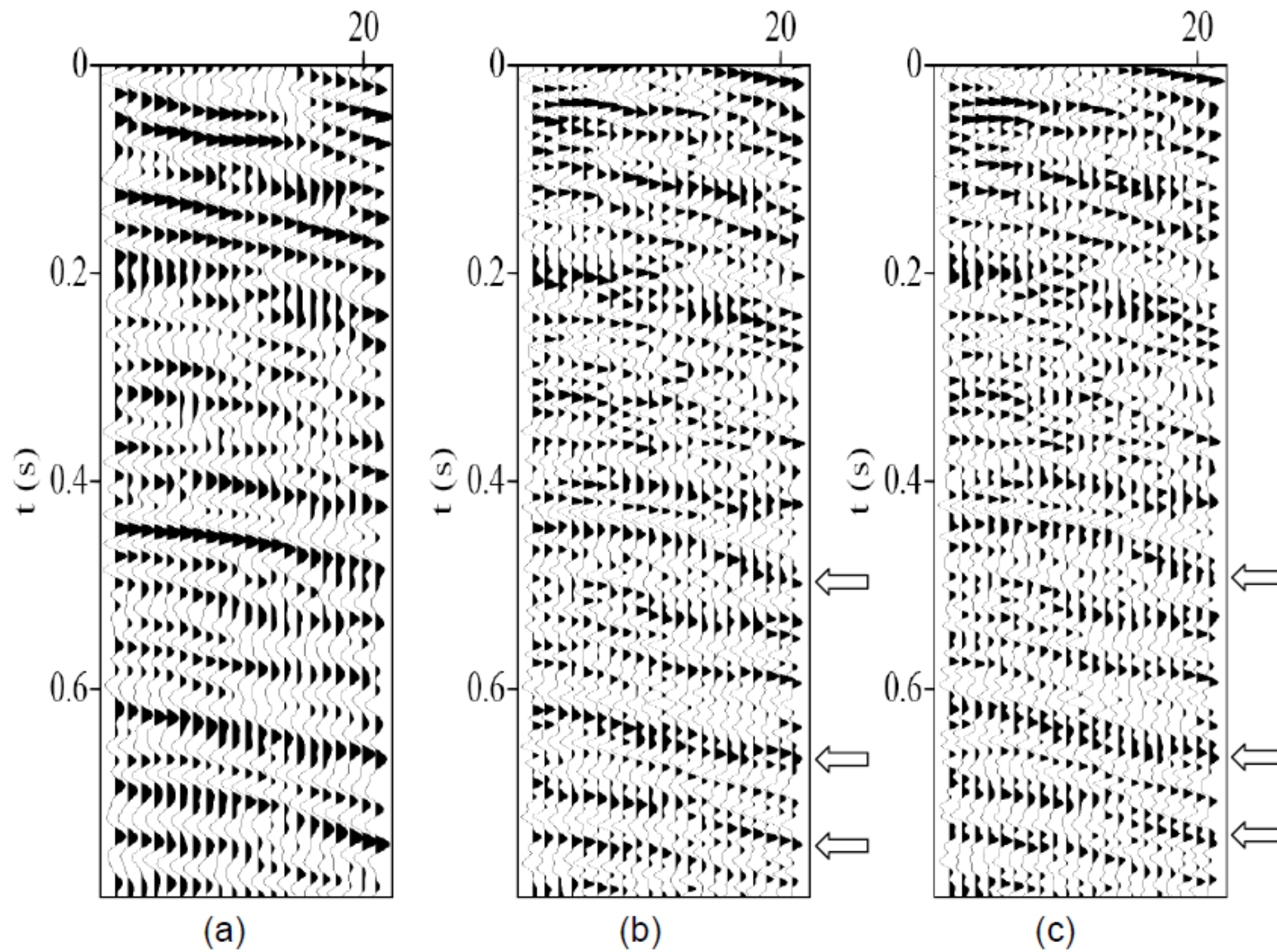
Figure 8. The real data example. (a) The shot gather. (b) Mixed phase deconvolution with a model length p = 4. (c) The minimum phase deconvolution.
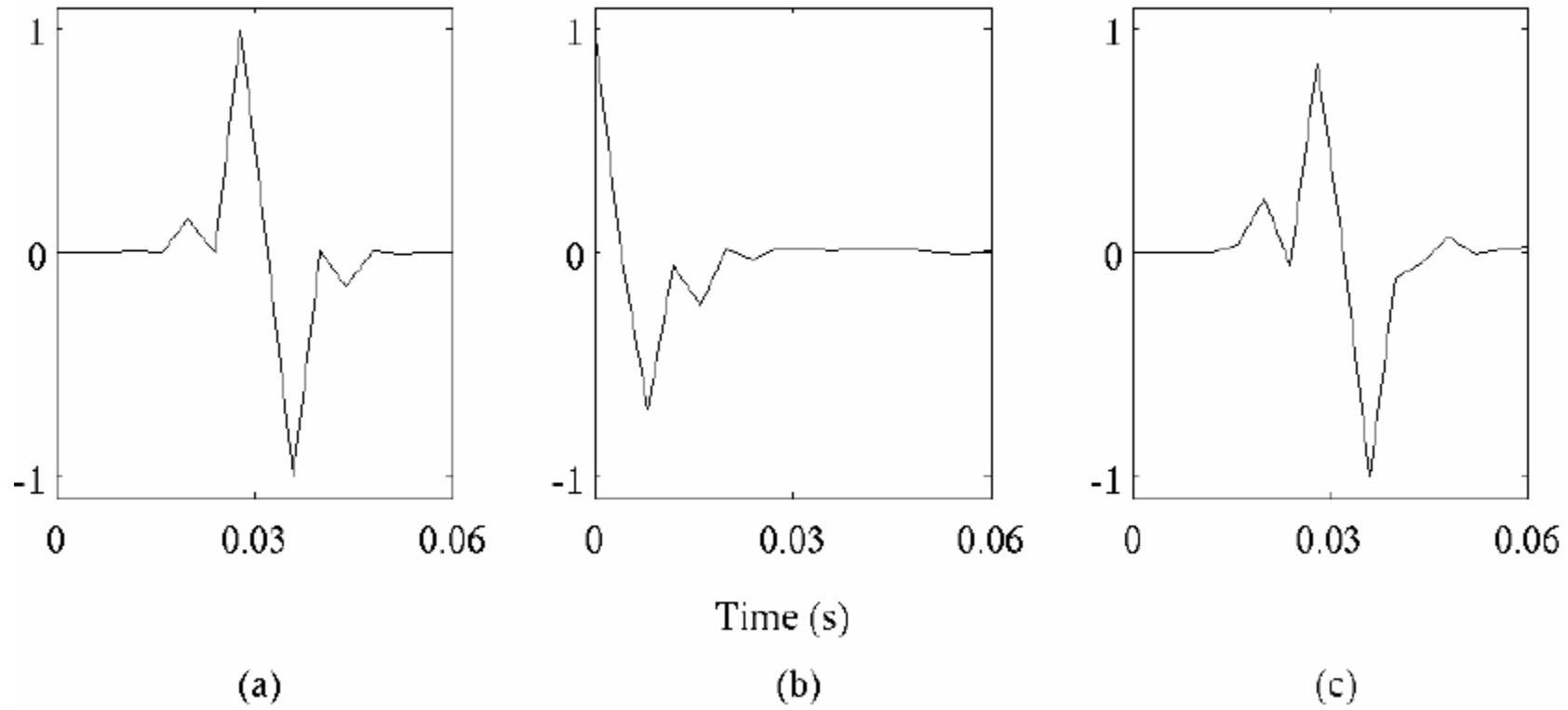
Figure 9. (a) The true 900 phase shifted wavelet. (b) The estimated minimum phase wavelet. (c) The recovered wavelet. Model length p = 4.