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Blind *L*1-norm Multichannel Deconvolution for Seismic Source Function Estimation

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Summary

In this work, similar to that proposed by Canadas (2002), synthetic data were used to analyze a blind deconvolution method to estimate source wavelets, which consists in alternating with single-channel reflectivity estimation, and multichannel wavelet estimation. The method is able to estimate the correct source wavelet under a wide range of conditions such as sparsity in the reflectivity series and band-width in the wavelet, and noise in the traces without making any a priori assumption of its phase.

Introduction

A seismic signal s_t can be represented as

$$s_t = w_t * q_t + n_t \qquad (1)$$

where * means convolution, w_t is the seismic wavelet, q_t is the reflectivity and n_t is the additive noise. The goal of deconvolution is, given s_t, to recover either q_t or w_t.

Conventional deconvolution methods depends on the previous knowledge of the properties of both reflectivity and wavelet; however, in this case we will not assum a previous knowledge about the wavelet phase (blind deconvolution).

Theory

Estimation of the wavelet given the reflectivity

Estimation of the wavelet when the reflectivity is known reduces to a multi-channel damped least-squares solution with quadratic regularization. The cost function from equation (1) is given by

$$\mathbf{J_w} = \frac{1}{N}\sum_{j=1}^{n} \| \mathbf{Q}_j\mathbf{w} - \mathrm{s}_j \|_2^2 + \mu \| \mathbf{w} \|_2^2 \qquad (2)$$

Where μ is the damping parameter and *j* is the number of traces. Minimizing equation (2) with respect to the unknown wavelet and equating it to zero we have that

$$\mathbf{w} = \left(\mathbf{R}_q + \mu\mathbf{I}\right)^{-1}\mathbf{g}_q \qquad (3)$$

where

$$\mathbf{R}_q = \frac{1}{N}\sum_{j=1}^{N} \mathbf{Q}_j^T \mathbf{Q}_j \tag{4}$$

is the average autocorrelation matrix of the reflectivities, and

$$\mathbf{g}_q = \frac{1}{N}\sum_{j=1}^{N} \mathbf{Q}_j^T \mathbf{s}_j \tag{5}$$

is the average cross-correlation vector of the traces with the reflectivities.

Estimation of the reflectivity given the wavelet

Estimation of the reflectivity when the wavelet is known involves defining as many cost functions as traces. Thus, from equation (1) we have that

$$\mathbf{J}_{\mathbf{q}_j} = \| \mathbf{W}\mathbf{q}_j - \mathbf{s}_j \|_2^2 + \beta R(\mathbf{q}_j), \qquad j = 1..., N. \tag{6}$$

In order to recover a sparse reflectivity sequence, the non-quadratic regularization term $R(\mathbf{q}) = \sum_t |q_t|$ and the associate hyperparameter β have been introduced in the cost functions.

$\sum_t |q_t|$ is known as the L_1-norm of **q** and has been studied by different authors (Taylor et. al., 1979; Levy and Fullegar, 1981; Oldenburg et. al., 1983; Santosa and Symes 1984). Minimizing equation (6) with respect to the unknown wavelet and equating it to zero we have that

$$\mathbf{q}_j = \left(\mathbf{W}^T\mathbf{W} + \beta\mathbf{\Lambda}(\mathbf{q}_j)\right)^{-1}\mathbf{W}^T\mathbf{s}_j, \qquad j = 1..., N. \tag{7}$$

where

$$\mathbf{\Lambda}(\mathbf{q})_{kk} = \frac{1}{|q_k| + \epsilon} \tag{8}$$

being ε an stabilization term.

Because equation (7) is non-linear, its solution must be found using an iterative approach like Iteratively Reweighted Least Squares (IRLS):

$$\mathbf{q}_j^{(k+1)} = \left(\mathbf{W}^T\mathbf{W} + \beta\mathbf{\Lambda}(\mathbf{q}_j^{(k)})\right)^{-1}\mathbf{W}^T\mathbf{s}_j, \qquad j = 1..., N. \tag{9}$$

Examples

The code was implemented statistically in a wide range of synthetic data set in order to test its stability.

Each data set was generated by convolving a particular wavelet with a particular reflectivity. Features of both wavelets and reflectivities used are described below:

Wavelets: trapezoidal amplitude spectrum wavelts with corner frequencies given by f=[f1,f2,f3,f4] were used. Frequencies f1 and f2 remained constant at f1=1 and f2=2 Hz while f3 and f4 were varied in the values of [20,30,40,60] and [30,40,50,70] [Hz] respectively, where f_3 < f_4. In each combination of f 3 and f4, a constant phase of c=[0,10,20,30,45,60] [deg] was applied.

Reflectivity: The code is very sensible to the number of traces that is chosen. After experiencing with different N number of traces, convergence is achieved for N > 10. In this work, data set consisted of 20 uncorrelated traces of nt=100 samples per trace. The amplitude of the reflector has a non-Gaussian distribution with sparsity δ varying in a range of [0.1, 0.3, 0.6, 0.8]. Sparsity defines the fraction of non-zeros reflectors in the reflectivity series.

Data was contaminated with white Gaussian noise with both signal to noise ratio of 5 and 20. Kurtosis was used to calculate the phase correction that was applied to the estimated wavelet. The maximum of the Kurtosis indicates the correct phase correction needed to convert the estimated wavelet in a zero phase wavelet. Figures 1 and 2 show examples of the performance of the algorithm under different values of SNR, sparsity and frequency.

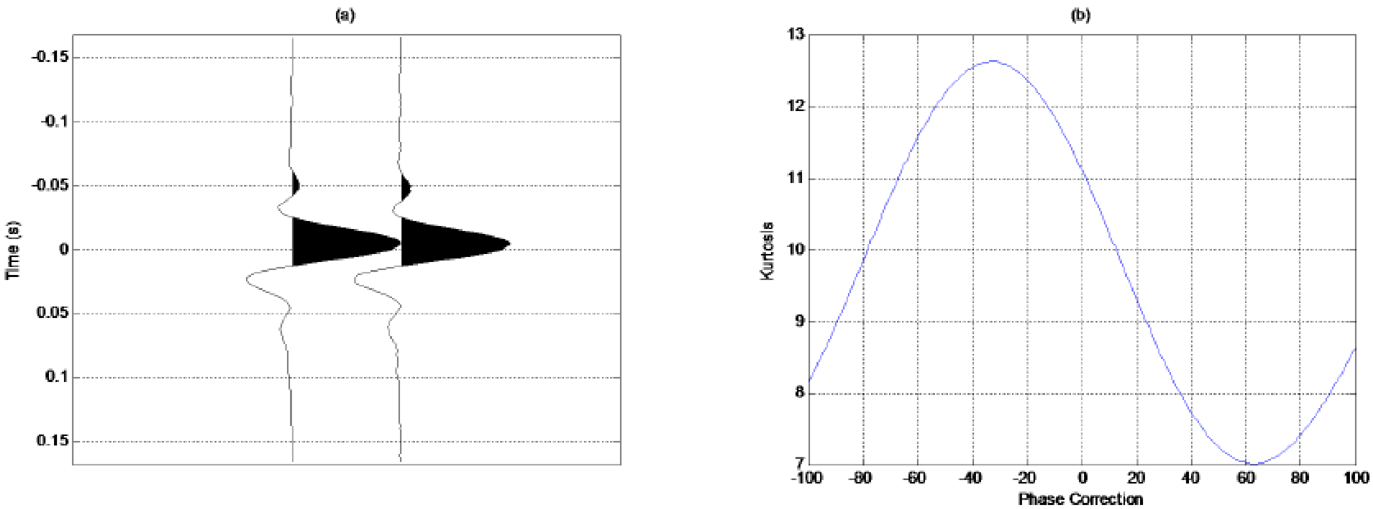

Figure 1. Example of the performance of the code when SNR=20, sparsity=0.8, and constant phase added of c=30: (a) true wavelet (left) and estimated wavelet (right), (b) kurtosis vs phase correction. Maximum of the kurtosis corresponds to a phase correction of -33 deg.

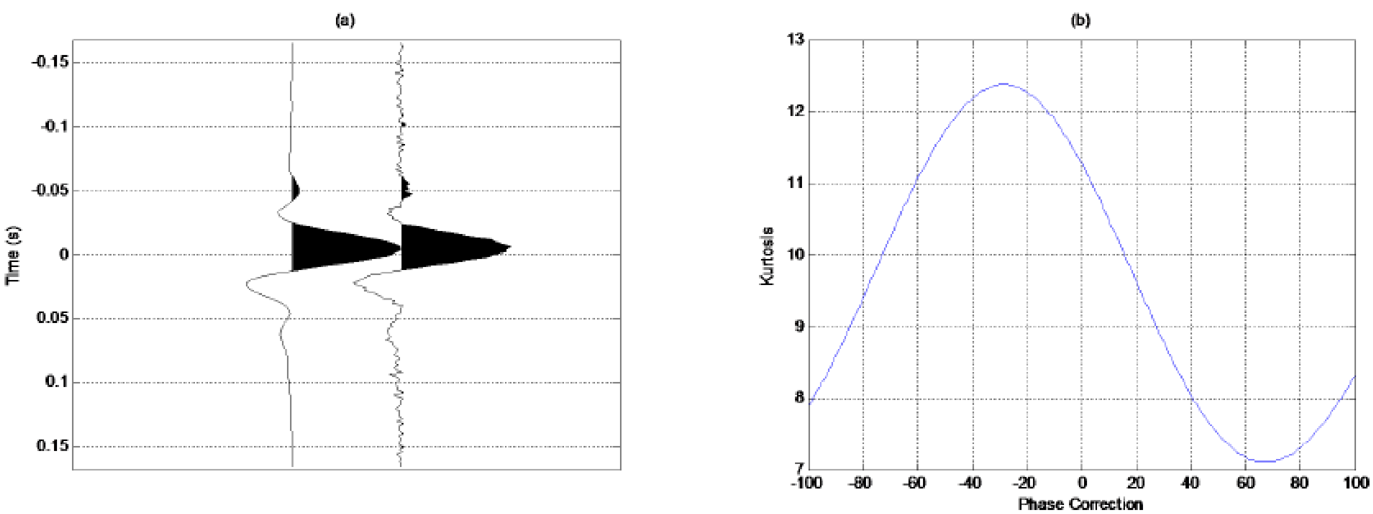

Figure 2. Example of the performance of the code when SNR=5, sparsity=0.1, and constant phase added of c=30: (a) true wavelet (left) and estimated wavelet (right), (b) kurtosis vs phase correction. Maximum of the kurtosis corresponds to a phase correction of -29 deg.

Conclusions

The assumption of non-correlated traces on which this code was developed is fundamental to guarantee the adequate performance of this method. The quality of the result also depends on choosing sufficient number of seismic traces. The level of sparsity of the reflectivity series and the amplitude content of the seismic wavelet do not affect phase estimations. In summary, this code has the potential to be used in real data that shows a complex geology.

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