Velocity model building with weighted linearized inversion – a VSP data case study

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Summary
First arrival traveltimes inversion of downgoing wave from Vertical Seismic Profiling (VSP) provides more localized estimation of in situ anisotropy than surface seismic. However, when first arrival data is contaminated by colored noise, conventional inversion that is based on least squares may not give the correct solution. In this paper, we address this problem with a data weighting inversion technique and apply it to a real case study.

Introduction
Using VSP data to build a velocity model has become a basic routine in VSP data processing. As a pre-processing step, the recorded wave field is separated into downgoing and upgoing wave fields. The recorded first arrivals of downgoing waves that directly propagate from sources located on the surface of the earth and received by sensors at different depths in a well, contain important information related to the propagating medium, which can be used to invert a velocity model within the VSP observation area, e.g. White et al., 1983; Gaiser, 1990; Miller and Spencer, 1994; Grecha and Mateeva 2007. Usually, even though there are errors in first arrival picking, linear least squares inversion is used as the tool to determine the velocity model assuming that errors have a Gaussian distribution. Another way to reduce the uncertainties of the data is by using a higher order polynomial fitting as a filter to remove data errors (e.g.Kumar and Hornby, 2011). This also requires the assumption of a Gaussian distribution of errors. However, in real cases, such an assumption of Gaussian error distribution may not be valid and therefore, the inverted velocity model may be not an unbiased estimate of the true model. In this paper, we will show a real case where data errors are not due to random picking errors but come from other different unknown factors: e.g. mis-organized sensors locations, shot depths, etc.. To obtain the best estimate of an earth model, the characteristics of the errors need to be carefully examined to obtain some a priori information for data quality and when this a priori information is applied to the inversion, the result can be significantly improved.

Data description
The input data comes from a 3D VSP survey carried out in Western Canada. Figure 1 displays the geometry of this 3D VSP survey. The data set has 1375 shot locations with approximate shot intervals of 5m and shot line intervals of 20 m. The maximum offset is 420 m from the wellhead. A 148 level VSP tool and another 50 level VSP tool were used to cover the 9.85 – 304.35 m depth interval of a vertical borehole with a nominal effective spacing of 1 m above 100m depth and then 2 m below 100 m depth. The VSP data, sampled at 1 ms, have a frequency range up to 400 Hz. Figure 2 is an example of the oriented component(Hmax’ ) that is used for the first arrival pick. This oriented component is a result of rotating the vertical component (Z) and oriented horizontal component (Hmax) to maximize the P-wave energy of the first arrivals onto a single component. This effectively points the receiver at the source.
Thus, all down-going P-wave energy is contained in this oriented component. The oriented horizontal component (Hmax) is the result of rotating horizontal X and Y components. P-wave energy in the horizontal plane is maximized along the first arrivals of this component (Ron. Hinds etc. 1996). The first arrival pick is based on an interactive module and is, therefore, a combination of automatic and manual processing. To see the quality of the first arrival data, a higher order polynomial fit is used to generate reference data provided that the area to be investigated can be represented by a simple layered velocity model. The misfit between the original first arrival and the fitted data can be used to define the standard deviations of data errors.

Figure 1: Plane view of the 3D VSP geometry  
Figure 2: Oriented component with first arrival picks

Figure 3 shows an example of the deviation of a receiver gather at the depth of 274 m and it is displayed as a colored distribution of the errors. The amplitude of errors goes from -8.4 ms to 11.2 ms. Figure 4 shows the histogram for the deviation and it is obviously far from a normal Gaussian distribution. We tried various methods to correct these errors but because of the lack of information for source locations, we were not successful.

Figure 3: Deviation between the real and high order polynomial filtered first arrival time  
Figure 4: Deviation from mean value of absolute misfit for receiver gather at 274 m depth

**Basic inverse theory**

Given observed data, with a first arrival traveltime, $b^{obs}$ our problem is to estimate an earth velocity model where the prediction error between modeled data and observed data is minimal in a least squares sense, i.e.
\[ E = \min_{m} |b^{obse} - b|^2 \]
\[ Gm = b \]  
where \( b \) is a predicted traveltime and \( G \) is called the data kernel matrix. Usually, equation (1) belongs to a mixed-determined problem and a constraint of the measure of solution simplicity is imposed, i.e.
\[ L = [m - \langle m \rangle]^T W_m [m - \langle m \rangle] \]  
We can also apply a weight matrix \( W_e \) that defines the relative contribution of each individual error to the total prediction error, i.e.
\[ E = e^T W_e \epsilon \]  
Then, the solution of the estimated model can be written as (e.g. Menke, 1982)
\[ m^{est} = \langle m \rangle + [G^T W_e G + \epsilon^2 W_m]^{-1} G^T W_e [d - G \langle m \rangle] \]  
where \( \epsilon \) is a parameter chosen to yield a solution that has a reasonably small prediction error.

The weighted measure of the prediction error plays a very important role in our data inversion. As analyzed before, the first arrival data shows that some observations have more accuracy than others. In this case one would like the prediction error \( e_i \) of the more accurate observations to have a greater weight in the quantification of the overall error \( E \) than the inaccurate observations.

**Initial model (\( m \)) selection**
In equation (1), a good initial model can guarantee a successful inversion because the relationship between the travel time and velocity model is not linear while equation (1) is a linearized inversion. The initial model is a 1D velocity model that is inverted by near offset data inversion. Because offset is small the ray paths are all almost vertical straight lines. With the constraints of the well log to set thicknesses of layers, the inverted interval velocity can be largely reliable.

**Weight matrices**
Two weight matrices need to be constructed before inversion with equation (1). The model weighting matrix \( W_m \) is chosen as a unit matrix. We choose this because all a priori information has been used in the initial model parameters estimate and hence we believe it to be a good model. Therefore, the term \( \epsilon^2 W_m \) in equation (1) works as a stability factor, i.e. damping factor. However, matrix \( W_e \) needs to be chosen carefully. First, considering that the errors which most affect the result are all individually independent, we choose a diagonal matrix \( W_e \). Each element on the diagonal corresponds to each of the observed data points and we assign a relative weighting to it. The amplitude of each element selection is based on the following considerations:
First, we calculate the mean value of the receivers’ absolute misfit between the real first arrival time and the first arrival time predicted by the initial velocity model for each shot line. The shot lines whose absolute mean receiver misfits are bigger than a specified threshold value are given a small weight. This threshold value is chosen based on the absolute mean value of the misfit of all shot lines. Then, for each receiver gather of each shot line, we calculate the mean value of the same misfit as the first step. The shot whose misfit is bigger than a threshold value is given another small weight. This threshold value is chosen based on the absolute mean value of the misfit of this shot line. Finally, for each data point, we multiply the weights of the first step and the second step and use this multiplication result as the final weight for this data point.

**Results**
In order to show the significance of the data weighting matrix in inversion, we put the data into inversion with and without data weights, which are shown in Figures 5 and 6, respectively. In Figure 5, the events
near the edges of the image are scattered and the events are not very continuous especially at around 250 m depth. Comparing Figures 5 and 6 we see more continuous events and the edges of the image are obviously improved.

**Conclusion**

Use of a higher order polynomial fit to clear the noise and outliers in the first arrival times has the assumption that the noise is random or Gaussian. When this assumption is not valid, the resulting velocity model can be misleading. A weighted linearized velocity inversion method is proposed. The weight for each data point is given by evaluating the misfits between the data first arrival time and the predicted first arrival time using a reliable initial velocity model. This method is applied to a 3D VSP data acquired in Western Canada area. The final results show that this weighted linearized velocity inversion provides a more accurate velocity model.

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**References**


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