AAPG Search and Discovery Article #90187 © CSPG/CSEG/CWLS GeoConvention 2013, Integration: Geoscience Engineering Partnership, 6-12 May 2013, Calgary, AB, Canada

A joint LSPSM/Inversion Algorithm For High Resolution Time Lapse Imaging

Abdolnaser Yousefzadeh* and John C. Bancroft, CREWES, University of Calgary, Calgary, Alberta, Canada
*ayousefzad@yahoo.com

Summary

Similarity of the data acquisition geometry of the baseline and monitor surveys is an essential key for a successful time lapse imaging. Different acquisition geometries of the seismic surveys produces different patterns of acquisition footprints. The resulting time lapse image will show these differences which may dominate the reflectivity changes in the reservoir due to the production or injection of fluids. A joint least squares Kirchhoff prestack migration (LSPSM) of both baseline and monitor data is introduced which attenuates the migration artifacts and returns high resolution LSPSM and time lapse images.

Introduction

Time lapse is the study of seismic data recorded in the same area at different times to measure changes in the physical properties of the hydrocarbon reservoirs or injection sites. Hydrocarbon production or gas/solvent injection may greatly change the physical properties of the reservoir rocks. Physical changes must be significant enough to be tractable from their seismic responses. Ignoring the effect of different environmental noise, near surface effects, and processing procedures, a key point in comparing two seismic surveys is that both, baseline and monitor surveys, have identical acquisition geometries. However, this is not always possible due to technical limitations or cost. Streamer feathering in marine data acquisition may causes same problem. The effect of streamer feathering is more significant for far offset receivers.

Consider a baseline seismic experiment as

$$\boldsymbol{d_0} = \boldsymbol{G_0}\, \boldsymbol{m_0}, \tag{1}$$

where $\boldsymbol{d_0}$, $\boldsymbol{G_0}$, and $\boldsymbol{m_0}$ are the recorded data, forward modeling operator, and reflectivity of the baseline survey, respectively. Suppose the Earth's reflectivity changes from $\boldsymbol{m_0}$ to $\boldsymbol{m_1}$ after a period of time as:

$$\boldsymbol{m_1} = \boldsymbol{m_0} + \boldsymbol{\Delta m}, \tag{2}$$

where $\boldsymbol{m_1}$ is the reflectivity at the time of the monitor surveying, and $\boldsymbol{\Delta m}$ is the difference in reflectivity between two data acquisitions. The monitor survey records data, $\boldsymbol{d_1}$, which is expressed by

$$\boldsymbol{d_1} = \boldsymbol{G_1}\, \boldsymbol{m_1}, \tag{3}$$

where $\boldsymbol{G_1}$ is the forward modeling operator of the monitor survey. Kirchhoff migration of these two data sets will produce migration images which are different, not only due to changes in the reflectivity model, but also different acquisition artifacts (Ayeni and Biondi, 2010; Yousefzadeh and Bancroft, 2012).

Replacing the conventional Kirchhoff migration with Kirchhoff LSPSM can attenuate acquisition footprints and create a high resolution seismic image (Nemeth et al., 1999). Separate damped LSPSM of the baseline and monitor surveys is an effective method to reduce the acquisition artifacts and to make the final images comparable. The advantages of using LSPSM and data reconstruction methods for time lapse studies of seismic data was discussed by Yousefzadeh (2013). Yousefzadeh (2013) performed a separate LSPSM for each baseline and monitor data sets, ignoring the presence of the other data set (Yousefzadeh and Bancroft, 2012; Yousefzadeh, 2013). Here, we show the simultaneous inversion of the baseline and monitor survey data using a joint inversion algorithm.

Joint inversion of time lapse data by LSPSM

Ayeni and Biondi (2010) performed least squares joint inversion of time lapse data using two related formulations. Their first formulation dealt with the simultaneous inversion of multiple images, referred to as the "Regularized Joint inversion of Multiple Images" (RJMI), the second formulation dealt with the inversion of the baseline and the difference (time lapse) images, referred to as the "Regularized Joint inversion for Image Differences" (RJID). The RJMI method returns baseline and monitor images, and the output of the RJID method is the baseline image and differences between the baseline and monitor image. Ayeni and Biondi (2010) used approximations to the wave equation least squares Hessian matrices to perform their inversion in the image domain. Implementing least squares in the image domain enabled them to target-orient their equation and reduce the high cost of the wave equation least squares migration.

In this study, we use Kirchhoff LSPSM for the joint inversion of time lapse data which is cheaper than the wave equation methods. To do so, with the same methodology that Ayeni and Biondi (2010) implemented, we combine the two cost functions for the separate damped LSPSM/inversion of the baseline survey

$$J_0(\boldsymbol{m_0}) = \|\boldsymbol{G_0 m_0} - \boldsymbol{d_0}\|^2 + \mu_0^2\|\boldsymbol{m_0}\|^2, \tag{4}$$

and the monitor survey

$$J_1(\boldsymbol{m_1}) = \|\boldsymbol{G_1 m_1} - \boldsymbol{d_1}\|^2 + \mu_1^2\|\boldsymbol{m_1}\|^2 , \tag{5}$$

where μs are the tradeoff parameters, into the Multiple Image Joint Inversion (MIJI) cost function as

$$J_{MIJI}(\boldsymbol{m_0}, \boldsymbol{m_1}) = \left\| \begin{bmatrix} \boldsymbol{G_0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G_1} \end{bmatrix} \begin{bmatrix} \boldsymbol{m_0} \\ \boldsymbol{m_1} \end{bmatrix} - \begin{bmatrix} \boldsymbol{d_0} \\ \boldsymbol{d_1} \end{bmatrix} \right\|^2 + \left\| \begin{bmatrix} \mu_0 & 0 \\ 0 & \mu_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{m_0} \\ \boldsymbol{m_1} \end{bmatrix} \right\|^2. \tag{6}$$

This is similar to RJMI. The time lapse image can be computed by

$$\boldsymbol{\Delta m_{miji}} = \boldsymbol{m_{miji1}} - \boldsymbol{m_{miji0}}, \tag{7}$$

where $\boldsymbol{m_{miji0}}$ and $\boldsymbol{m_{miji1}}$are the baseline and monitor survey reflectivity images resulting from MIJI, respectively (Yousefzadeh and Bancroft, 2012).

Alternatively, joint inversion can be performed to obtain a time lapse image, $\boldsymbol{\Delta m}$, directly by minimizing the Image Difference Joint Inversion (IDJI) cost function as

$$J_{IDJI}(\boldsymbol{m_0}, \boldsymbol{\Delta m}) = \left\| \begin{bmatrix} \boldsymbol{G_0} & \boldsymbol{0} \\ \boldsymbol{G_1} & \boldsymbol{G_1} \end{bmatrix} \begin{bmatrix} \boldsymbol{m_0} \\ \boldsymbol{\Delta m} \end{bmatrix} - \begin{bmatrix} \boldsymbol{d_0} \\ \boldsymbol{d_1} \end{bmatrix} \right\|^2 + \left\| \begin{bmatrix} \mu_0 & 0 \\ 0 & \mu_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{m_0} \\ \boldsymbol{\Delta m} \end{bmatrix} \right\|^2. \tag{8}$$

This is similar to the RJID formulation of Ayeni and Biondi (2010).

We used the least squares conjugate gradients method (LSCG) (Scales, 1987) to minimize these cost functions. As an example, ignoring regularization terms, $\mu_0 = \mu_1 = 0$, a simplified LSCG algorithm to minimize J_{IDJI} is shown in Table 1. As seen in this table, each iteration in this LSCG algorithm includes performing two migrations and two modelings. Therefore, each iteration is at least four times as expensive as one time migration of the baseline survey data. Tests showed that the convergence of the above mentioned joint inversion algorithms were slower than the convergence of separate LSPSM of the baseline and monitor survey data.

We performed MIJI and IDJI on a synthetic model when the interval velocity of the dipping layer at 1 s drops by 15% from 4000 m/s at baseline survey to 3400 m/s at the monitor survey (Figure 1). Results of MIJI are shown after 50 LSCG iterations in Figure 2. The resulting time lapse image from MIJI is shown in Figure 3. Figures 4a and 4b show the baseline and time lapse images resulting from IDJI, respectively. Comparison between Figure 3 and Figure 4b shows that a higher resolution time lapse is achieved by MIJI. This seems to be due to the difference between the structure of the Hessian matrices in the MIJI and IDJI. The IDJI forward modeling matrix is 50% denser than the MIJI matrix. The MIJI matrix is symmetric where the IDJI matrix is not. Therefore, MIJI is better solved by the LSCG method. It is necessary to mention that a time lapse image of MIJI is obtained after 50 LSCG iterations which is equal to the cost of 200 conventional migrations.

Table 1: LSCG algorithm for solving the image difference joint inversion equation.

$\boldsymbol{m}_0 = \text{an initial guess or } \boldsymbol{m}_0 = \boldsymbol{0}$

$\boldsymbol{\Delta m} = \boldsymbol{0}$

$\boldsymbol{s}_0 = \boldsymbol{d_0} - \boldsymbol{G_0}\boldsymbol{m}_0$

$\boldsymbol{s}_1 = \boldsymbol{d1} - \boldsymbol{G_1}(\boldsymbol{m}_0 + \boldsymbol{\Delta m})$

$\boldsymbol{r}_1 = \boldsymbol{G}_1^{\boldsymbol{T}}\boldsymbol{s}_1$

$\boldsymbol{r}_0 = \boldsymbol{G}_0^{\boldsymbol{T}}\boldsymbol{s}_0 + \mathbf{r_1}$

$\boldsymbol{p}_0 = \boldsymbol{r}_0$

$\boldsymbol{p}_1 = \boldsymbol{r}_1$

$\boldsymbol{q}_0 = \boldsymbol{G_0}\boldsymbol{p}_0$

$\boldsymbol{q}_1 = \boldsymbol{G_1}(\boldsymbol{p_0} + \boldsymbol{p_1})$

for $i = 0$: iterations limit

$$\alpha_{i+1} = \frac{\boldsymbol{r_{0i}} \,.\, \boldsymbol{r_{0i}} + \boldsymbol{r_{1i}} \,.\, \boldsymbol{r_{1i}}}{\boldsymbol{q_{0i}} \cdot \boldsymbol{q_{0i}} + \boldsymbol{q_{1i}} \cdot \boldsymbol{q_{1i}}}$$

$$\boldsymbol{m_0}_{i+1} = \boldsymbol{m_0}_i + \alpha_{i+1}\boldsymbol{p_0}_i$$

$$\boldsymbol{\Delta m}_{i+1} = \boldsymbol{\Delta m}_i + \alpha_{i+1}\boldsymbol{p_1}_i$$

$$\boldsymbol{s_0}_{i+1} = \boldsymbol{s_0}_i - \alpha_{i+1}\boldsymbol{q_0}_i$$

$$\boldsymbol{s_1}_{i+1} = \boldsymbol{s_1}_i - \alpha_{i+1}\boldsymbol{q_1}_i$$

$$\boldsymbol{r_1}_{i+1} = \boldsymbol{G}_{\boldsymbol{1}}^{\boldsymbol{T}}\boldsymbol{s_1}_{i+1}$$

$$\boldsymbol{r_0}_{i+1} = \boldsymbol{G}_{\boldsymbol{0}}^{\boldsymbol{T}}\boldsymbol{s}_{i+1} + \boldsymbol{r_1}_{i+1}$$

$$\beta_{i+1} = \frac{\boldsymbol{r_0}_{i+1} \,.\, \boldsymbol{r_0}_{i+1} + \boldsymbol{r_1}_{i+1} \,.\, \boldsymbol{r_1}_{i+1}}{\boldsymbol{r_1}_i \,.\, \boldsymbol{r_1}_i}$$

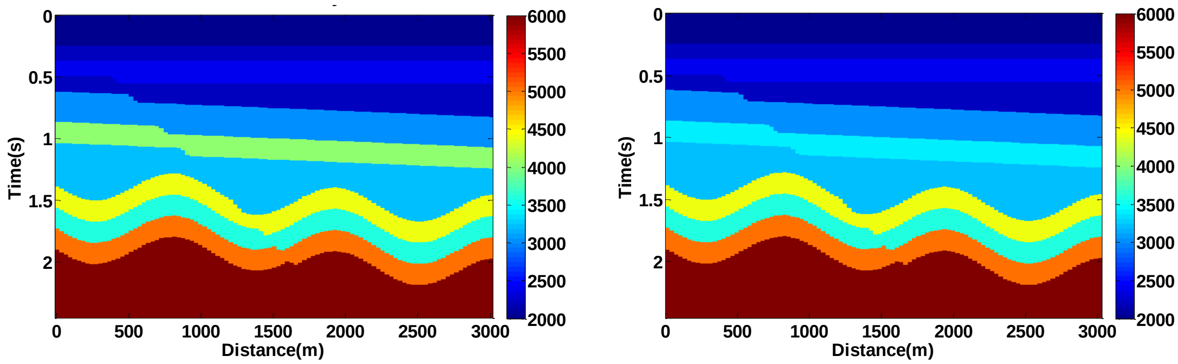
$$\boldsymbol{p_0}_{i+1} = \boldsymbol{r_0}_{i+1} + \beta_{i+1}\boldsymbol{p_0}_i$$

$$\boldsymbol{p_1}_{i+1} = \boldsymbol{r_1}_{i+1} + \beta_{i+1}\boldsymbol{p_1}_i$$

$$\boldsymbol{q_0}_{i+1} = \boldsymbol{G_0}\boldsymbol{p_0}_{i+1}$$

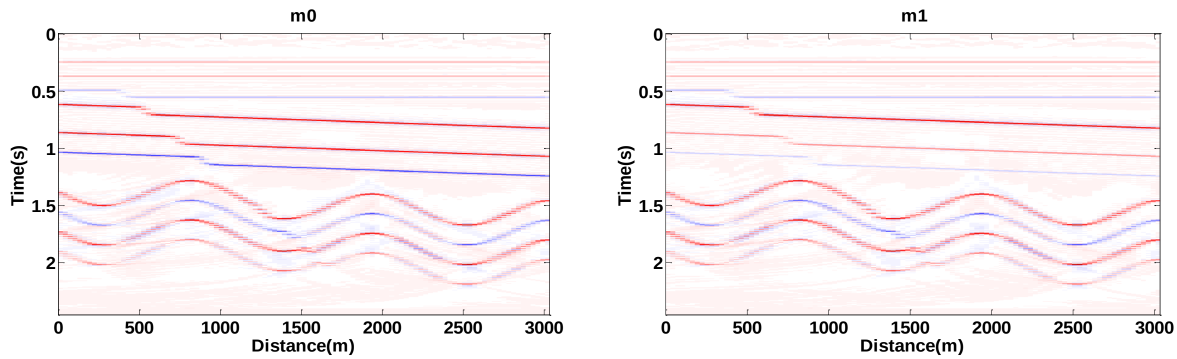
$$\boldsymbol{q_1}_{i+1} = \boldsymbol{G_1}(\boldsymbol{p_0}_{i+1} + \boldsymbol{p_1}_{i+1})$$

endfor



a) b)

Figure 1: Velocity model for a) baseline, and b) monitor survey.



a) b)

Figure 2: Reflectivity of a) baseline survey and b) monitor survey achieved by MIJI.

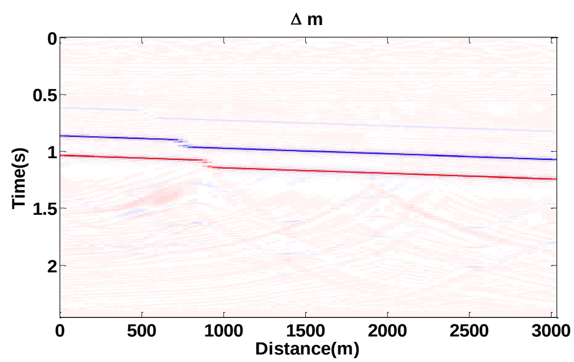
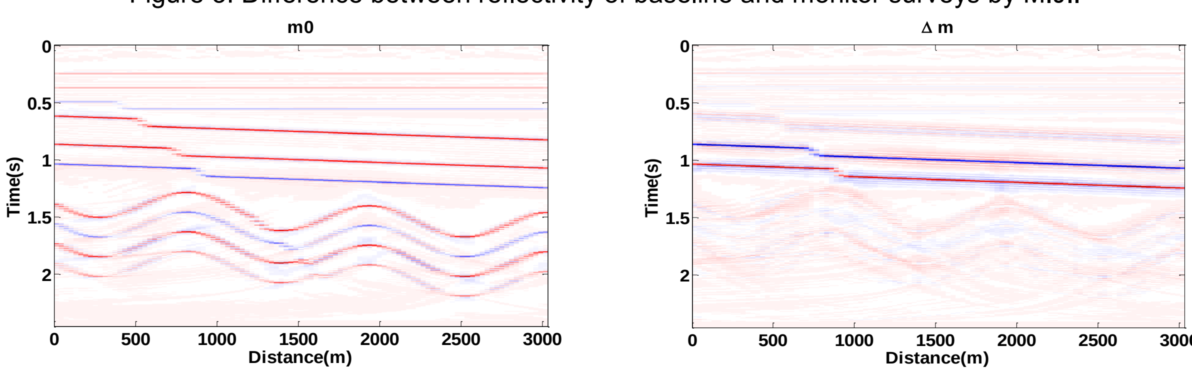


Figure 3: Difference between reflectivity of baseline and monitor surveys by MIJI.



a) b)

Figure 4: a) Reflectivity of baseline survey, and b) Time lapse image from by IDJI.

The formulation for the joint inversion of the baseline and monitor survey data can be extended to the joint inversion of a baseline and several monitor surveys. For example, if the subsurface reflectivity model changes from $\boldsymbol{m_1}$ to $\boldsymbol{m_2} = \boldsymbol{m1} + \boldsymbol{\Delta m_1}$ between the two monitor surveys, the second monitor survey data, $\boldsymbol{d_2}$, can be expressed by

$$\boldsymbol{d_2} = \boldsymbol{G_2}\,\boldsymbol{m_2} = \boldsymbol{G_2}(\boldsymbol{m_1} + \boldsymbol{\Delta m_1}) = \boldsymbol{G_2}(\boldsymbol{m_0} + \boldsymbol{\Delta m} + \boldsymbol{\Delta m_1}), \tag{9}$$

where $\boldsymbol{G_2}$ is the forward modeling operator of the second monitor survey. Then, the corresponding joint inversion methods can retrieve the $\boldsymbol{m_0}$, $\boldsymbol{m_1}$, and $\boldsymbol{m_2}$ via MIJI or $\boldsymbol{m_0}$, $\boldsymbol{\Delta m}$, and $\boldsymbol{\Delta m_1}$via IDJI by minimizing the following cost functions,

$$J_{MIJI}(\boldsymbol{m_0}\,,\boldsymbol{m_1},\boldsymbol{m_2}) = \left\|\begin{bmatrix}\boldsymbol{G_0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{G_1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{G_2}\end{bmatrix}\begin{bmatrix}\boldsymbol{m_0}\\ \boldsymbol{m_1}\\ \boldsymbol{m_2}\end{bmatrix} - \begin{bmatrix}\boldsymbol{d_0}\\ \boldsymbol{d_1}\\ \boldsymbol{d_2}\end{bmatrix}\right\|^2 + \left\|\begin{bmatrix}{}_0 & 0 & 0\\ 0 & {}_1 & 0\\ 0 & 0 & {}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{m_0}\\ \boldsymbol{m_1}\\ \boldsymbol{m_2}\end{bmatrix}\right\|^2, \tag{10}$$

and

$$J_{IDJI}(\boldsymbol{m_0}\,,\boldsymbol{\Delta m}\,,\boldsymbol{\Delta m_1}) = \left\|\begin{bmatrix}\boldsymbol{G_0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{G_1} & \boldsymbol{G_1} & \boldsymbol{0}\\ \boldsymbol{G_2} & \boldsymbol{G_2} & \boldsymbol{G_2}\end{bmatrix}\begin{bmatrix}\boldsymbol{m_0}\\ \boldsymbol{\Delta m}\\ \boldsymbol{\Delta m_1}\end{bmatrix} - \begin{bmatrix}\boldsymbol{d_0}\\ \boldsymbol{d_1}\\ \boldsymbol{d_2}\end{bmatrix}\right\|^2 + \left\|\begin{bmatrix}{}_0 & 0 & 0\\ 0 & {}_1 & 0\\ 0 & 0 & {}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{m_0}\\ \boldsymbol{\Delta m}\\ \boldsymbol{\Delta m_1}\end{bmatrix}\right\|^2. \tag{11}$$

Each iteration in the LSCG method to solve equation 10 or equation 11 is at least six times more expensive than the migration of the baseline data. This means that 20 iterations of LSCG to invert a time lapse study with two monitor surveys are 40 times more costly than the migration of all data sets.

Conclusion

A joint Kirchhoff LSPSM/inversion method for high resolution time lapse imaging is introduced. Assuming no difference other than different geometries in the acquisition and processing of the baseline and monitor surveys, it was shown how joint LSPSM of baseline and monitor data sets can attenuate acquisition footprints and creates reliable time lapse images. Formulations of the joint inversion of time lapse data to invert for the baseline image and time lapse image by the LSCG method are derived. It is important to mention that this is an expensive procedure. Each iteration of the joint inversion costs more than four times that of performing a single conventional migration. Other corrections such as removing the near surface effects and attenuating multiples must be performed before inverting the data.

Acknowledgements

Authors wish to acknowledge the sponsors of the CREWES project for their continuing support. They also appreciate the computer technicians of the CREWES project. Thanks to Rolf Maier in particular for his careful and attentive editing.

References

Ayeni, G., and B. Biondi, 2010, Target-oriented joint least-squares migration/inversion of time-lapse seismic data sets: Geophysics, **75**, R62-R73.

Nemeth, T., C. Wu, and G. T. Schuster, 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, 208-221.

Scales, J., 1987, Tomographic inversion via the conjugate gradient methods: Geophysics, **52**, 179-185.

Yousefzadeh, A., 2013, High resolution seismic imaging using least squares migration: PhD dissertation, University of Calgary.

Yousefzadeh, A., and J. C. Bancroft, 2012, LSPSM/inversion for pre- and poststack time lapse studies: CREWES Annual Research Report, Vol. **24**.