

Simultaneous P-P and P-S waveform inversion algorithm using Pre-Stack time imaging method

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Summary

Full Waveform Inversion (FWI) is ultimate goal of the seismic method. There have been enhancements on the different aspects of the approach mostly at the laboratory scale, but it has not gained much traction within the industry, mainly because of its huge computational time.

The conventional approach of elastic FWI requires a forward modeling and a depth migration. The forward modeling engine, which is usually based on a finite difference solution of the elastic wave equation, computes the data residual that compares the data derived from the current model with the real data from the true model. Migration is an adjoint operator of the forward modeling, which is usually based on Reverse Time Migration (RTM) on the data residual and finds the gradient function from the current model toward the true model.

Assuming multiple free data and smooth lateral variation of subsurface properties, this work serves as an introduction to elastic waveform inversion using Pre-Stack Time Migration (PSTM) and the corresponding forward modeling.

We present the result of simultaneous inversion on synthetic data and a field data example to show the robustness of the method.

Introduction

Seismic FWI was introduced by Tarantola (1984) to estimate high resolution subsurface properties from waveform information contained in seismic data. FWI is a least-squares approach to minimize the differences between synthetic and observed data during the updating of the model parameters.

Several authors have undertaken FWI in the time domain based on the finite difference solution to elastic waveform inversion. To study the long history of FWI the reader is referred to Virieux and Operto (2009) and Sears et al (2007).

This work is based on the theoretical framework of Tarantola (1984) who showed that classical Kirchhoff migration and corresponding forward modeling can be used in the FWI procedure. Our effort is to reduce the computational costs associated with gradient calculation and data prediction. In other work, the use of forward Pre-Stack time Kirchhoff operator for the prediction of P-P data from the reflectivity function (i.e. Schneider, 1978, Bleistein et. al., 2001) and the corresponding PSTM migration for the inversion process in FWI algorithm (Khaniani et. al., 2012) have been described.

Using the elastic inversion schemes that usually require the inversion of the model parameters in depth, we have developed an algorithm that predicts the mode converted P-S wave using the velocity in time. The algorithm is based on a scatter point response in P-wave reflection intercept time using Double Square Root (DSR) equation and the result of Zoeppritz solvers (Aki and Richards, 1980) from the scatter point to produce P-S data. This facilitates the updating process using the parameters of traveltimes and amplitudes of scatter point.

The methodology is fast compared to corresponding depth migration techniques in forward and inverse iterations however, since we are doing time migration, we are limited to models with moderate complexity. In this work, we assume the data are multiple free because of limitation in the forward and adjoint operator to handle the multiple data.

Theory and Method

Beylkin and Burridge, 1990) derived the equations of the perturbed waves in the shot records due to the perturbed elastic properties of the medium. From the perturbed waves, Tarantola (1984,1994, 1996) proposed the least squares criterion for the elastic inversion

$$S = \frac{1}{2} \left(\|U - U_{true}\|^2 \right), \quad (1)$$

Here, U represents the data predicted by the model parameters m (i.e., P-wave impedance, ρC^P , S-wave impedance, ρC^S and density ρ). He proposed the application of the steepest descent algorithm for the inversion of the model parameters

$$m(x)_{n+1} = m(x)_n - \alpha \gamma_n(x), \quad (2)$$

which minimizes equation (1). His suggested algorithm includes the following two main steps:

1. Forward modeling for calculation of data residuals
2. Depth migration of data residuals for gradient calculation $\gamma_n(x)$. This step is done by applying an “imaging principle” with forward propagation of the source and back propagation of the data in time.

This work is based on the generalized solution of Tarantola (1984), but contains forward modeling and migration of scatter point responses using the time migration algorithm. In this approximation the associated traveltimes are calculated from the Root Mean Square (RMS) velocity. The amplitude function is obtained using an estimation of the reflectivity function obtained from the Zoeppritz solvers (Aki and Richards, 1980). Assuming lower amplitude of S-P and S-S data from the injected source in z-component, in this work, the forward operator is designed only for reflection data of P-P and P-S data. In the forward modeling, the traveltime for P-P data is approximated by

$$t^{PP} \approx \phi^P(s, x) + \phi^P(x, r) = \frac{r(s, x)}{v_{RMS}^P} + \frac{r(x, r)}{v_{RMS}^P}, \quad (3)$$

and the traveltime for the P-S data is obtained by

$$t^{PS} \approx \phi^P(s, x) + \phi^S(x, r) = \frac{r(s, x)}{v_{RMS}^P} + \frac{r(x, r)}{v_{RMS}^S}. \quad (4)$$

To perform the waveform inversion, we need to implement both traveltime and amplitude considerations during forward modeling and migration of both P-P and P-S waves. Therefore, for the migration operator as the adjoint of the forward operator, an algorithm is designed to consider the common traveltime for the scatter point, $\phi^P(s, x)$. Then, at each scatter point, the algorithm sums the hyperbola corresponding to t^{PP} and t^{PS} for P-P and P-S data respectively. Hence, the migration operators for P-P and P-S data maps the data to the model space, τ , in the same time coordinate. This facilitates computation of the gradient function for P-P and P-S data.

Synthetic data example

In order to demonstrate the potential of the inversion technique, the true, smoothed starting and inverted velocity of P- and S- waves of a 1D synthetic geologic model is shown in Figures (1a & 1b). The second layer has no change in P-wave velocity but, an increase in S-wave velocity. The developed simultaneous inversion scheme considers the mode converted P-S waves that take place at interfaces. The dominant frequencies of the used minimum wavelets are 5-13 Hz. As shown in Figure (1b), both P-wave velocity and S-wave velocity inversions performed well, recovering the main features of the model including low impedance contrast of the S-wave in the second layer.

Field data example

Figure (2a) shows the migration result of a field vertical component (P-P dominant) of 51 shot records acquired by Nexen Inc in Canadian NE-BC region. To learn about the data acquisition parameters and geological settings of the study area the reader is referred to Zuleta (2012)'s MSc thesis. For a comparison, as shown in Figure (2b), the modeled shot records were migrated with the same migration algorithm used for field P-P data in Figure (2a). Figure (2c) shows the radial component (P-S dominant) data of the same shot records migrated using t^{PS} defined in equation (4). The velocity for migration is obtained by well log information from a location near the study area and was found to have smooth lateral variation in velocity. The shot records were modeled using the same velocity. The small difference in time of the migrated sections is due to the different static correction datum used during the processing.

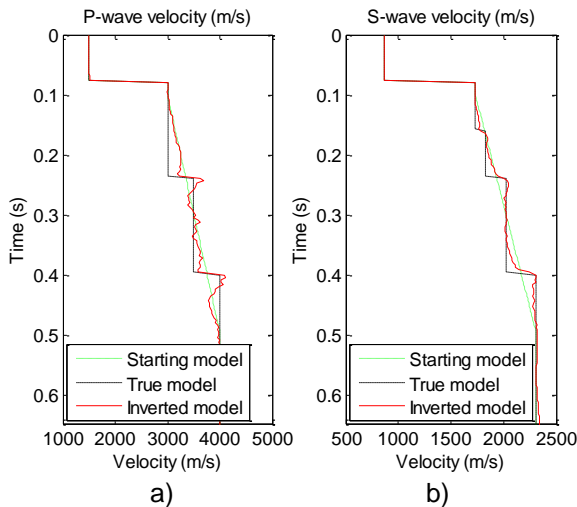


Figure 1: Simultaneous inversion result of a synthetic model using (5-13Hz) P-P and mode converted P-S data. a) P- wave velocity and b) S-wave velocity.

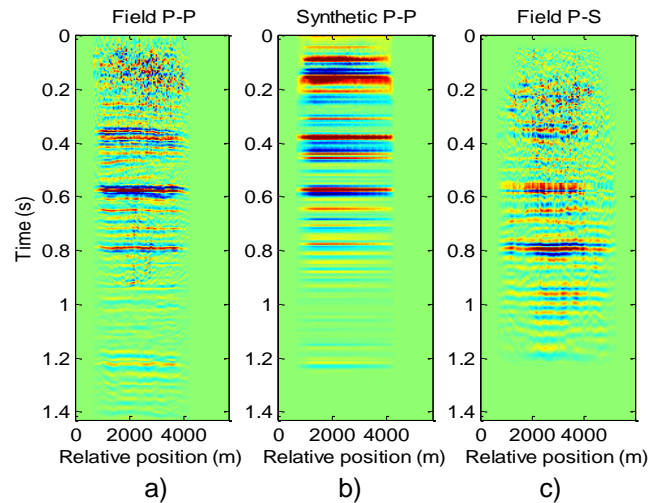


Figure 2: Comparison of Migration result in P- time $\phi(s, x)$. a) Migrated field P-P data b) migrated synthetic P-P data c) migrated field P-S data

The procedure of waveform inversion is based on the linearization of seismic reflection data. Therefore, in the field data we will have to eliminate the noise and preserve the original amplitude in order to minimize the objective function defined by equation (1). Noises in this case, are any signals that are not reflected waves and are not included in the forward modeling scheme. Examples of noise are multiple data, surface waves, surface noise and dead traces.

The preliminary simultaneous inversion result of P-wave and S-wave velocity inversion is shown in Figures (3a & 3b). The initial velocity for inversion is obtained using a linear increment (green dotted line) of the well log (solid blue curve) contained in Figures (3a & 3b). We inverted data for deeper reflectors that appeared beyond 0.5s on the vertical component (assumed to be P-P dominant) and

0.3s for radial component (P-S dominant). this was because our migration algorithm was more efficient in migrating the deeper reflectors.

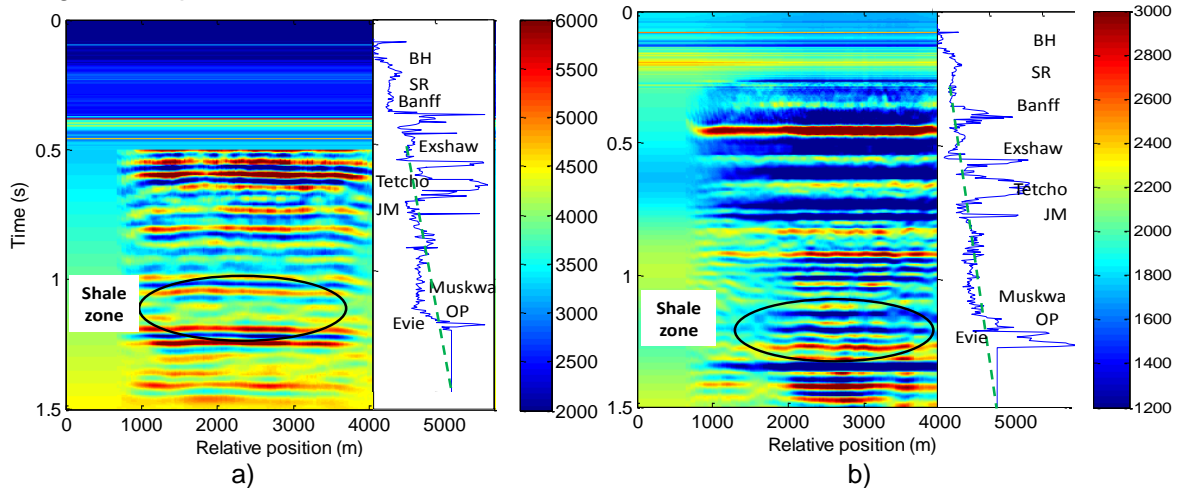


Figure 3: Inversion results of a) P- wave velocity, b) S- wave velocity correlated with overlaying sonic well log (solid blue curve). The frequency range of iterative inversion for P-P data is 5 -13 Hz and for P-S data is 5-9 Hz. Initial model is shown as green dotted line. The color scale is velocity.

Seismic data contains band limited frequencies; therefore we restricted the inversion frequencies from 5Hz to 13Hz. To improve the P-S inversion result, the low frequency components of the well log were added to the gradient function in order to update the S- wave velocity. Still, the limited frequency content of the source and non-linearity of the inverse problem caused ill-posedness for the inversion result. Consequently, as shown by the color scale in Figure (3), we had to display the inversion result within the range of 2000 to 6000 m/s for the P- wave and 1200 to 3000 m/s for the S- wave velocity inversion. However, the resulting inversions show good correlation with the well log data overlain on the Figure (3). An additional feature to note is the lateral variation of the inverted velocities within the ellipse corresponding to the shale formations of the Muskwa and Otter Park (OP). These results highlight the value in employing waveform inversion for the extraction of subsurface physical properties and their lateral variations.

Conclusions

In case, where the geological structure has small lateral velocity variations, the linearized solution of the seismic reflection inverse problem can be obtained using the Pre-Stack Time Migration (PSTM) and corresponding forward modeling. It requires updating the velocity in time and it incorporates accurate diffraction stack weighting of the PSTM data.

We have developed an algorithm that performs waveform inversion on the mode converted P-S data using both traveltime and amplitude information within shot domain data. We have used the Zoepritz solvers for amplitudes and the Double Square Root (DSR) equation for traveltime consideration of P-S and P-P data during the waveform inversion.

The simultaneous use of travel time and amplitude in the wavefield analysis using PSTM provides another effective tool to an improved understanding of shale gas reservoirs for unconventional resource development and extraction.

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