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AVO theory for large contrast elastic and anelastic targets in pre-critical regimes

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Summary

Interpretable, easy-to-calculate corrective terms are added to first order (i.e., Aki-Richards type) approximate solutions of the Zoeppritz equations. The corrections quantitatively and qualitatively account for the influence of large contrasts and target anelasticity in the precritical regime. The formulation permits several observations pertinent to AVO to be made, regarding

1. The non-negligible importance of target V_P to mode conversions, i.e., the θ dependence of R_{PS};

2. The importance of the number $V_P/V_S=2$ to all elastic reflection coefficients;

3. Wave scattering from contrasts in Q_P and Q_S alone; and

4. The relationship between reciprocal quality factors and the frequency rate of change of anelastic reflection coefficients.

The basics of the approach are summarized and the above points are illustrated with AVO curves calculated from plausible large contrast elastic and anelastic media.

Introduction

We consider the problem of AVO analysis (Castagna and Backus, 1993; Foster et al., 2010) when the contrast causing the reflection in question is large, and when the target medium is potentially anelastic. Both of these situations, alone and/or in combination, are outside the scope of standard linearized AVO theory. In this section we will discuss the two issues in turn, and outline our approach for modelling AVO curves under these circumstances.

AVO for large contrasts

When choosing tools for modeling AVO in media with large contrasts, we face a dilemma. There are two main classes of tool:

1. Linearized reflection coefficient approximations such as those due to Aki and Richards (2002) (hereafter AR). These approximations permit quantitative calculation of $R(\theta)$, are relatively stable when inverted, and are expressed, conveniently, in terms of relative parameter changes as opposed to absolute values (Stolt and Weglein, 1985). Finally, the approximations are straightforward to analyze, readily supplying qualitative insight into the data-parameter relationship. However, their applicability fails, in both quantitative and qualitative domains, when contrasts grow. And,

2. The full Zoeppritz equations, which have the opposite attributes: from them we can calculate exact (plane wave) reflection coefficients, regardless of contrast. But, they deal in absolute parameter values, are notoriously unstable to invert, and are qualitatively opaque.

The dilemma is that little in the way of middle ground between these two classes exists, and so when contrasts are large our available toolkit is diminished. To exemplify the problem, let us consider the AR approximation for the coefficient associated with P-S mode conversion:

$$R_{PS} \approx -k\left[(1+\delta)\frac{\Delta\rho}{\rho} + 2\delta\frac{\Delta V_S}{V_S}\right], \qquad (1)$$

where k and δ are functions of incidence angles and average medium properties, and the ratios $\Delta V_S / V_S$ and $\Delta\rho / \rho$ measure the change in V_S and density across the reflecting boundary (e.g., Stewart et al., 2002). With this formula, values of R_{PS} can be directly calculated from input incidence and target medium properties. However, of equal importance is the ease with which certain statements can be made about the relationship between R_{PS} and the contrasts. For instance, what is the effect on R_{PS} of a change in the target V_P? None—there is no $\Delta V_P/V_P$ term in the approximation (though depending on how δ and k are defined there may be weak implicit dependence). This lack of dependence on V_P is difficult to discern from an inspection of the full Zoeppritz equations. The problem is, as contrasts grow, not only does the quantitative accuracy of formula (1) fail, but so does the qualitative accuracy of these kinds of statements. Consider Figure 1, in which exact R_{PS} curves for three large contrast targets are considered. The target V_P is different for each example, but otherwise the properties of the three media are identical. Linearly, we expect these curves to be identical, and yet, both at large angles (a) and small angles (b), we see significant differences. Though equation (1) is not wrong, *per se*, at these contrasts we have departed from its domain of accuracy.

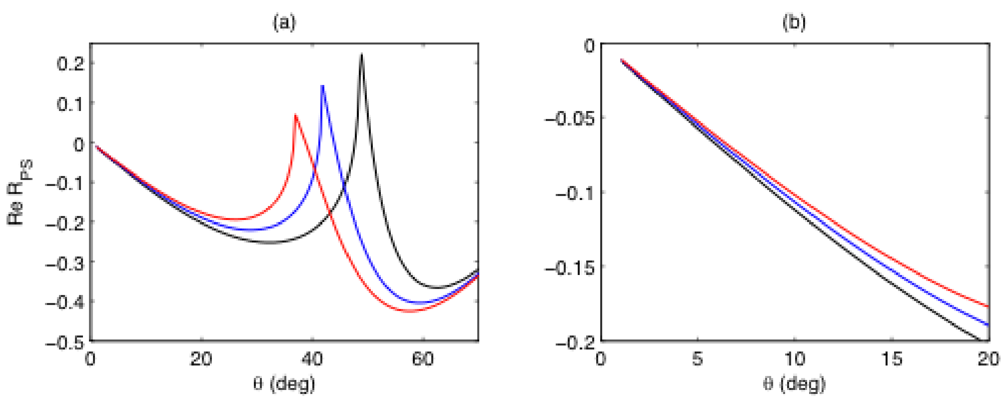

Figure 1: Dependence of R_{PS} curves on target V_P. Three curves are plotted with the same incidence parameters, ρ_0=2.0gm/cc, V_{P0}=3km/s and V_{S0}=1.5km/s, and the same target parameters ρ_1=2.5gm/cc, V_{S1}=2.3km/s. Target V_{P1} varies: black curve 4.0km/s, blue curve 4.5km/s and red curve 5.0km/s. In (a) the curve is plotted from 0 to 70 degrees; in (b) from 0 to 20 degrees.

Anelastic reflections

A potentially important second order influence on seismic AVO is target anelasticity, which, though far from a new idea (e.g., White, 1965; Kjartansson, 1979), has grown in interest in recent years (e.g., Stovas and Ursin, 2001; Chapman et al., 2006; Odebeatu et al., 2006; Lines et al., 2008; Behura and Tsvankin, 2009). Although fundamental questions remain about correctly formulating anelastic reflection coefficients (Krebes and Daley, 2007), the problem is on relatively solid theoretical footing (Borcherdt, 2009), and with simple alterations of the Zoeppritz equations, displacement reflection coefficients can be calculated for an elastic incidence medium overlying an anelastic target medium obeying a nearly-constant Q model. In Figure 2 we illustrate the potential importance of anelastic AVO/AVF (see also Innanen, 2011) with three R_{PP} curves for the same anelastic target at different fixed frequencies. Under the right circumstances, target Q can have a significant effect.

Objectives and approach

When the medium property contrast causing a reflection is sufficiently large, linearization error and target anelasticity (if it is present) become significant to AVO modeling. We seek corrections to be added to AR type approximations, which (1) return accuracy to pre-critical calculations while maintaining their qualitative interpretability, and (2) include the influence of target anelasticity. While such second order approximations have been developed in the past (Stovas and Ursin, 2003), the focus has tended to be on increased numerical accuracy; the forms we derive follow instead from our interest in both qualitative and quantitative predictions. Once determined, the corrective terms permit a set of observations and predictions to be made, regarding: (i) the importance of target V_P, (ii) the importance of a V_P/V_S ratio of 2, (iii) P-P reflections from contrasts in Q_P and Q_S only, (iv) mode conversions from contrasts in Q_P and Q_S only, and (v) the relationship between reciprocal Q values and the frequency rate of change of the associated reflection coefficients.

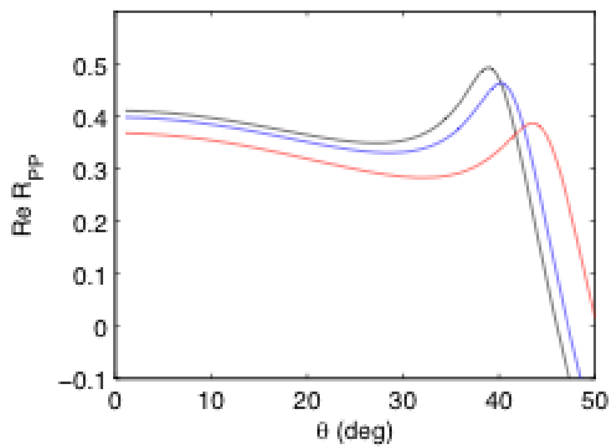

Figure 2: Importance of anelasticity to AVO. R_{PP} curves for incidence medium V_{P0}*=2.0km/s,* V_{S0}*=1.5km/s and* ρ_0*=2.0gm/cc*, and target medium V_{P1}*=3.2km/s,* V_{S1}*=1800km/s, =3.0gm/cc,* Q_{P1} *= 10, and* Q_{S1}*=20*, for three fixed frequencies: 100Hz (black), 40Hz (blue) and 5Hz (red).

The Zoeppritz equations in matrix form

In this section we lay out the Zoeppritz equations in a convenient matrix form, and then adapt them to admit anelastic target media. The matrix equation for a P-wave incident on an elastic target is due to Achenbach (1973), and has been used in more or less this form by several authors (e.g., Levin, 1986; Keys, 1989). The companion equation for an S-wave incident on an elastic target is also provided. The version we have derived can be shown to be consistent with the more complete and complex relations on pg. 140 of Aki and Richards (2002) and in Appendix A of Castagna and Backus (1993), though we use incidence angles. Consider a plane P-wave incident with angle θ on a horizontal interface, and a plane S-wave incident with angle φ on the same interface. We seek expressions for the four associated displacement reflection coefficients R_{PP}, R_{PS}, R_{SP}, and R_{SS}. Setting X = sinθ, Y = sinφ, and making use of the functions $\Gamma_j=\sqrt{1-j^2X^2}$, $\Gamma^j=1-2j^2X^2$, $\Gamma_j'=\sqrt{1-j^2Y^2}$ and $\Gamma^{j\prime}=1-2j^2Y^2$, the Zoeppritz equations may be written as

$$\mathbf{P}\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = \mathbf{b}_P, \quad \text{and} \quad \mathbf{S}\begin{bmatrix} R_{SS} \\ R_{SP} \\ T_{SS} \\ T_{SP} \end{bmatrix} = \mathbf{b}_S, \qquad (2)$$

where

$$\mathbf{P} = \begin{bmatrix} -X & -\Gamma_B & CX & \Gamma_D \\ \Gamma_1 & -BX & \Gamma_C & -DX \\ 2B^2X\Gamma_1 & B\Gamma^B & 2AD^2X\Gamma_C & AD\Gamma^D \\ -\Gamma^B & 2B^2X\Gamma_B & AC\Gamma^D & -2AD^2X\Gamma_D \end{bmatrix}$$

and

$$\mathbf{S} = \begin{bmatrix} -\Gamma_1 & -B'Y & \Gamma'_F & EY \\ Y & -\Gamma'_{B'} & FY & -\Gamma'_E \\ \Gamma'^1 & 2Y\Gamma'_{B'} & AF\Gamma'^F & 2AF^2Y\Gamma'_E \\ -2Y\Gamma'_1 & B'\Gamma'^1 & 2AF^2Y\Gamma'_F & -AE\Gamma'^F \end{bmatrix},$$

and where

$$\mathbf{b}_P \equiv \begin{bmatrix} X \\ \Gamma_1 \\ 2B^2X\Gamma_1 \\ \Gamma^B \end{bmatrix}, \quad \text{and} \quad \mathbf{b}_S \equiv \begin{bmatrix} \Gamma'_1 \\ Y \\ \Gamma'^1 \\ 2Y\Gamma'_1 \end{bmatrix}.$$

The incidence and target medium properties are contained in *A*-*F*:

$$A = \frac{\rho_1}{\rho_0}, \quad B = \frac{V_{S_0}}{V_{P_0}}, \quad B' = \frac{V_{P_0}}{V_{S_0}}, \quad C = \frac{V_{P_1}}{V_{P_0}},$$
$$D = \frac{V_{S_1}}{V_{P_0}}, \quad E = \frac{V_{P_1}}{V_{S_0}}, \quad \text{and} \quad F = \frac{V_{S_1}}{V_{S_0}}. \qquad (3)$$

To admit anelastic target media, we adjust V_{P1} and V_{S1} following Aki and Richards (2002), such that C and F become

$$C = \frac{V_{P_1}}{V_{P_0}}\left[1 - \frac{F_P(\omega)}{Q_{P_1}}\right], \quad F = \frac{V_{S_1}}{V_{S_0}}\left[1 - \frac{F_S(\omega)}{Q_{S_1}}\right], \qquad (4)$$

where

$$F_P(\omega) = \frac{i}{2} - \frac{1}{\pi}\log\left(\frac{\omega}{\omega_P}\right) \text{ and } F_S(\omega) = \frac{i}{2} - \frac{1}{\pi}\log\left(\frac{\omega}{\omega_S}\right), \quad (5)$$

and where ω is angular frequency, and ω_P and ω_S are reference frequencies. The quantities *D=F B* and *E = CB′* are affected also.

Solutions

Any one of the four displacement reflection coefficients can be determined from the above equations using Cramer's rule (Keys, 1989). Forming two auxiliary matrices $\mathbf{P}_P$ and $\mathbf{P}_S$ by replacing the first and then second columns of $\mathbf{P}$ with $\mathbf{b}_P$, and then forming a further two auxiliary matrices $\mathbf{S}_S$ and $\mathbf{S}_P$ by replacing the first and then second columns of $\mathbf{S}$ with $\mathbf{b}_S$, we have

$$R_{\mathrm{PP}} = \frac{\det\mathbf{P}_P}{\det\mathbf{P}}, \quad R_{\mathrm{PS}} = \frac{\det\mathbf{P}_S}{\det\mathbf{P}}, \quad R_{\mathrm{SS}} = \frac{\det\mathbf{S}_S}{\det\mathbf{S}}, \quad R_{\mathrm{SP}} = \frac{\det\mathbf{S}_P}{\det\mathbf{S}}. \quad (6)$$

To expand these solutions in series about the perturbations experienced by each of the five medium parameters across the interface, we define

$$\begin{aligned} a_{VP} &= 1 - V_{P_0}^2/V_{P_1}^2, \ a_{VS} = 1 - V_{S_0}^2/V_{S_1}^2, \ a_\rho = 1 - \rho_0/\rho_1, \\ a_{QP} &= Q_{P_1}^{-1}, \quad \text{and} \quad a_{QS} = Q_{S_1}^{-1}. \end{aligned} \quad (7)$$

Eliminating all target properties in equations (2) in favour of the above perturbations, and expanding the determinants in equations (6) in orders of these perturbations as well as *sinθ* and *sinφ*, we arrive at series expansions of R_{PP}, R_{PS}, R_{SP} and R_{SS}.

The linear terms of these expansions are consistent with AR type approximations, and the second order terms are corrections whose importance grows with target contrast. Third order and higher terms are available also, but, as their number and complexity grow rapidly with order, and since we do not wish to lose the qualitative interpretability of the approximations, we will neglect them in this paper.

In Figure 3 we illustrate the first and second order approximations for an anelastic target versus the exact solutions. The significant increase in accuracy, as well as remaining error (to be corrected with third order and higher terms), are both evident. In the next sections some examples of the predictions and observations emerging from this approach are provided. Because we truncate the expansions at low order in *sinθ* and *sinφ*, our analysis is restricted to pre-critical angles. This is justifiable as most AVO analysis occurs in this regime.

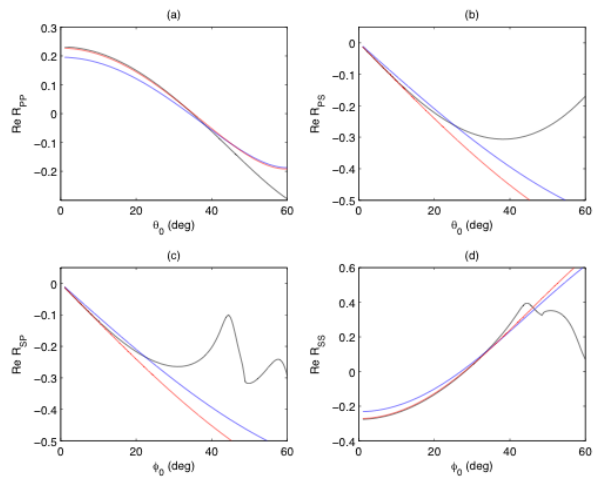

Figure 3: Exact (black), first order (blue) and second order (red) AVO curves for incidence medium V_{P0}*=2.0km/s,* V_{S0} *= 1.5km/s,* ρ_0*=2.0gm/cc*, and target medium V_{P1}*=2.2km/s,* V_{S1}*=1.8km/s,* ρ_1*= 3.0gm/cc,* Q_{P1}*=15, and* Q_{S1}*=10*. All four reflection coefficients are plotted: (a) R_{PP}, (b) R_{PS}, (c) R_{SP} and (d) R_{SS}.

Elastic AVO applications

Converted wave AVO and target V_P

Letting $Q_{P1}\rightarrow\infty$ and $Q_{S1}\rightarrow\infty$, expanding R_{PS} in series as outlined above, and retaining terms in *sinθ,* we determine first order terms of the R_{PS} approximation and their second order corrections:

$$R_{\rm PS}(\theta) \approx -\frac{V_{S_0}}{V_{P_0}}\sin\theta\, a_{VS} - \left(\frac{V_{S_0}}{V_{P_0}}+\frac{1}{2}\right)\sin\theta\, a_\rho + R_{\rm PS}^{(2)}, \qquad (8)$$

where

$$\begin{aligned} R_{\rm PS}^{(2)} &= \frac{1}{4}\left(\frac{V_{S_0}}{V_{P_0}}\right)\sin\theta a_{VP}\, a_{VS} \\ &+\frac{1}{4}\left(\frac{V_{S_0}}{V_{P_0}}-\frac{1}{2}\right)\sin\theta a_\rho (a_{VP}+a_{VS}) \\ &-\frac{3}{4}\left(\frac{V_{S_0}}{V_{P_0}}\right)\sin\theta a_{VS}^2 - \frac{1}{2}\sin\theta a_\rho^2. \end{aligned}$$

For small perturbations and small angles, the first two terms in equation (8) are consistent with AR (equation 1). Indeed, the two expressions are in qualitative agreement that target V_P is inconsequential to R_{PS}: there is no first order contribution from the a_{VP}. As we have seen (Figure 1b), however, the accuracy of this statement fails as contrasts grow; dependence of R_{PS} on target *VP* appears at all angles.

At second order all terms contain products of perturbations such as $a_{VP}\times a_{VS}$. They measure how relative changes in target properties couple in determining reflection strengths. The target V_P, via a_{VP}, does not alone influence R_{PS} at second order. However, by equation (8) we discern that target V_P does nevertheless have an influence, through coupling with density and V_S. In fact, since these terms are first order in *sinθ*, this influence is expected to be present at all nonzero angles. In Figures 4a–b, the quantitative results of including the second order corrections are illustrated for two of the examples from Figure 1. The second order corrections are valid for small angles only, as these expansions were truncated beyond first order in *sinθ*, but below 20° the true variations of R_{PS} with target V_P appear to be well captured by equation (8).

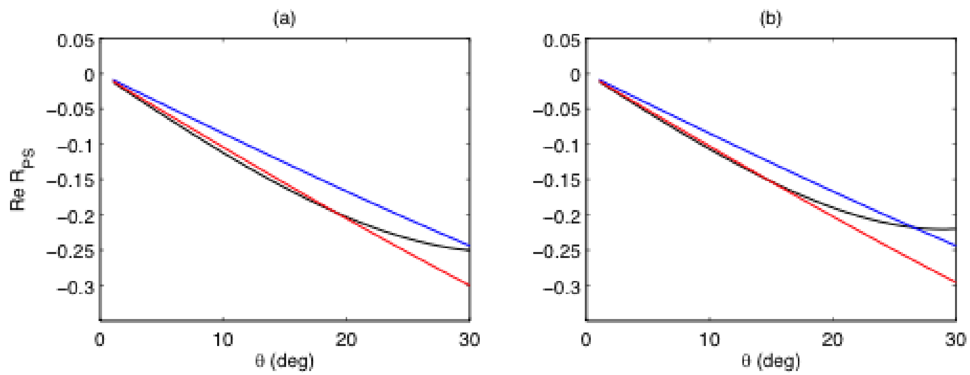

Figure 4: Exact (black), first order (blue) and second order (red) R_{PS} approximations for varying target V_{P1}. Incidence medium V_{P0}=3.0km/s, V_{S0} = 1.5km/s, ρ_0=2.0gm/cc, and target medium V_{S1}=2.3km/s, ρ_0= 2.5gm/cc, Q_{P1}=15, and Q_{S1}=10. Target V_{P1} (a) 4.0km/s, (b) 4.5km/s.

Coupling with density and the importance of VP/VS=2

To the Zoeppritz equations, V_P / V_S=2 is a special number. The specialness, which is connected to coupling between density and V_P & V_S, becomes particularly apparent at large contrast, when second order corrective terms of the type we are discussing in this paper are significant. By inspection, two of the five second order correction terms in equation (8) are proportional to the quantity

$$\left(\frac{V_{S_0}}{V_{P_0}} - \frac{1}{2}\right), \qquad (9)$$

which is zero when the V_P-V_S ratio in the incidence medium is 2. These two terms (which involve $a_\rho \times a_{VP}$ and $a_\rho \times a_{VS}$) have in common that they measure the coupling between density (a_ρ) and the other parameter changes (a_{VP} and a_{VS}). This is a persistent phenomenon. For all four of R_{PP}, R_{PS}, R_{SP}, and R_{SS}, all nonzero second order terms coupling a_ρ with other perturbations are proportional to this quantity. If V_{P0} /V_{S0}=2, these terms, which represent a full third of the elastic second order corrections, vanish, changing the way parameter contrasts can alter reflection coefficients, and dramatically simplifying the mathematical “landscape” of the Zoeppritz equations. AVO analysis is known to simplify when V_P /V_S=2. Furthermore, in discussions of these matters it is often apparent that there is a connection with density. For instance, in deriving their *fluid line,* Foster et al. (2010) explain that V_P/V_S= 2 simplifies the slope of R_{PP} vs. $sin^2\theta$.

From the point of view of the current formulation, these two facts are “explained” as being due to the parameter coupling between density and all other parameters. If there is no density contrast, there is no coupling between V_P, V_S and ρ, trivially. However, the less trivial case in which there are strong density contrasts, but V_{P0}/V_{S0}=2,is seen to also correspond to a decoupling of the density contrasts with those of V_P and V_S.

Anelastic applications

P-P reflections from Q_P and Q_S contrasts

Lines et al. (2008) have discussed the possibility of reflections from targets across which *Q* alone undergoes a contrast. This possibility has been confirmed in laboratory environments (Bourbie and Nur, 1984; Lines et al., 2012), and though the latter results may raise as many interesting questions as they answer, we can confirm that the expansion of R_{PP} for anelastic targets is in agreement with their basic observation. Truncating the anelastic approximation for R_{PP} beyond first order, and simulating a contrast in Q_P and Q_S only by setting $a_{VP} = a_{VS} = a_\rho = 0$, we obtain

$$R_{\mathrm{PP}}(\theta,\omega) \approx \left[4\left(\frac{V_{S_0}}{V_{P_0}}\right)^2 F_S(\omega)\sin^2\theta\right] a_{QS} - \left[\frac{F_P(\omega)}{2\cos^2\theta}\right] a_{QP}, \quad (10)$$

confirming that to first order, Q_P and Q_S contrasts can cause a P-P reflection. We may further confirm that either Q_P or Q_S can, alone, create a P-P reflection, though this may be primarily of mathematical interest, as Q_P and Q_S co-vary for most real Earth materials (e.g., Waters, 1978). In Figure 5a, an example for plausible incidence and target medium properties is illustrated. Whether the average discrepancy constitutes a true anomaly is therefore uncertain. However, we point to peaks in the red curve, which are present in both components, and which are unlikely to be matched by the blue curve regardless of scale factor, as representing avenues for further investigation.

Mode conversions from Q_S and Q_P contrasts

Continuing in the vein of “reflections from *Q* contrasts”, the current formulation also allows us to predict the generation of mode conversions from targets across which quality factors alone undergo a contrast. Truncating the anelastic R_{PS} approximation beyond first order and again setting $a_{VP}=a_{VS}=a_\rho=0$, we have

$$\begin{aligned} R_{\mathrm{PS}}(\theta,\omega) \approx & \left[2\left(\frac{V_{S_0}}{V_{P_0}}\right) F_S(\omega)\sin\theta\right] a_{QS} \\ & + \left[\left(\frac{V_{S_0}}{V_{P_0}}\right) F_P(\omega) F_S(\omega)\sin\theta\right] a_{QS} \times a_{QP}. \end{aligned} \quad (11)$$

From this we may conclude that an elastic incidence medium, which is suddenly interrupted by a contrast in Q_S alone, can cause a mode conversion to first order. A similar contrast in Q_P cannot, to first order, cause such a conversion, but if Q_P and Q_S both undergo a contrast, the former can influence the amplitude of the conversion at second order. This latter influence should be expected to be minimal, except for contrasts in which the target Q_P and Q_S are both very small. In Figure 5b, an example for plausible incidence and target medium properties is illustrated.

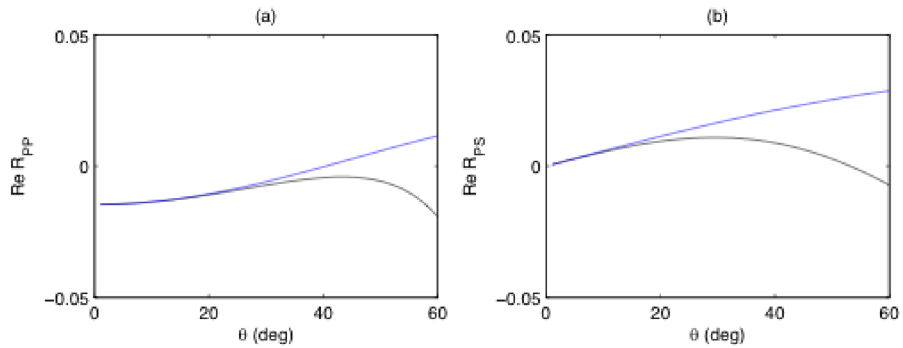

Figure 5: Exact (black) and first order approximations (blue) of reflections from contrasts in Q_P and Q_S: (a) R_{PP}, and (b) R_{PS}. Medium properties $V_{P0}=V_{P1}=2.0km/s$, $V_{S0}=V_{S1}=1.5km/s$, and $\rho_0=\rho_1=2.0gm/cc$. Target $Q_{P1} = 15$ and $Q_{S1} = 10$.

Q_P and Q_S and the frequency rate of change of R_{PP}, R_{PS} and R_{SS}

Anelastic AVO provides several modes for inversion, in particular for target Q estimation (Innanen, 2011). The reflection coefficients associated with an anelastic boundary are in general complex and frequency dependent. Within the model we have adopted, Q_P and Q_S alone are responsible for the frequency dependence of R_{PP}, R_{PS}, R_{SP}, and R_{SS}. Therefore, to first order, differences of anelastic reflection coefficients across frequency are sensitive to variations in these parameters, and insensitive to V_P, V_S and ρ. We expand R_{PP}, R_{PS}, and R_{SS} as discussed above and truncate beyond first order in all five anelastic perturbations. Taking derivatives of these coefficients with respect to frequency extinguishes the influence of a_{VP}, a_{VS} and a_ρ. Thereafter it is straightforward to prove that the reciprocal quality factors in the target medium are given by

$$Q_P^{-1} \approx (2\pi \times \omega)\frac{\partial}{\partial\omega}R_{PP}(0,\omega), \qquad (12)$$

$$Q_S^{-1} \approx -\left(\frac{\pi}{2} \times \frac{V_{P_0}}{V_{S_0}} \times \frac{\omega}{\sin\theta}\right)\frac{\partial}{\partial\omega}R_{PS}(\theta,\omega), \qquad (13)$$

$$Q_S^{-1} \approx -[2\pi \times \omega \times (1-7\sin^2\phi)^{-1}]\frac{\partial}{\partial\omega}R_{SS}(\phi,\omega). \qquad (14)$$

The two Q_S formulas in equations (13)–(14) have realizations for every angle of data available. In contrast, we have fixed $\theta = 0$ for the P-wave Q_P case, as under these conditions the formula is helpfully simplified. In Figure 6a-b, the recovery of Q_S = 10 using equations (13)–(14) is illustrated. Linearization error grows with decreasing frequencies, where the perturbation is larger because of its dependence on dispersion.

Conclusions

A simple-to-use truncated expansion of the anelastic Zoeppritz equations adds a small number of second order corrective terms to Aki- Richards type linearizations. These corrective terms expose the coupling between contrasts in V_P, V_S and ρ (and Q_P, Q_S if desired) in determining the four elastic/anelastic reflection coefficients. Because of the qualitative interpretability of the corrections, we may arrive at some theoretical justification for several “large contrast, low angle” AVO phenomena, such as: (1) the importance of target V_P to mode conversions, (2) the importance of the number V_P/V_S= 2, (3) reflections and mode conversions from contrasts in Q_P and Q_S only, and (4) inverse relations between Q_P, Q_S and the frequency rate of change of anelastic reflection strengths.

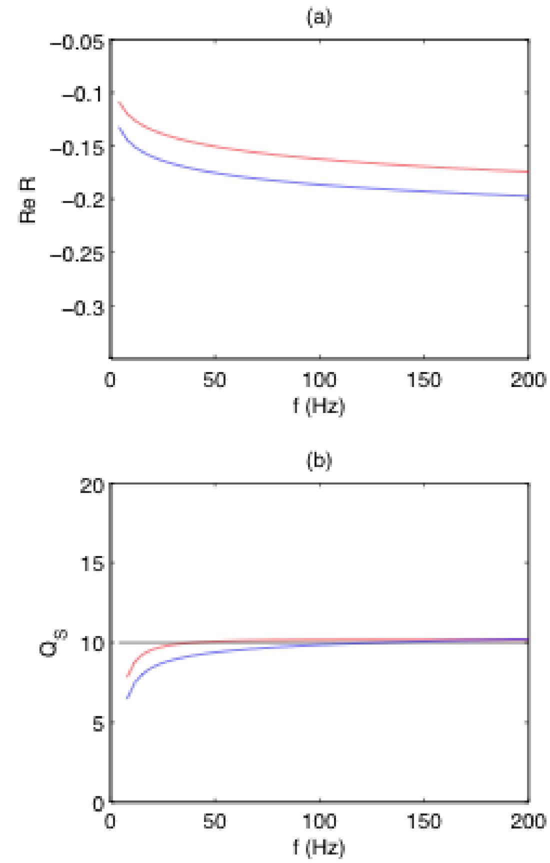

Figure 6: Recovery of target Q_S through calculation of rate of change of R with frequency. (a) Real parts of R_{PS} and R_{SS} (red and blue respectively) as functions of frequency. (b) Calculation of scaled derivatives in equations (13)-(14) in comparison with the actual Q_S value. Medium properties $V_{P0}=2.0km/s$, $V_{P1}=2.2km/s$, $V_{S0}=1.5km/s$ and $\rho_0=2.0gm/cc$ and $\rho_1=2.5gm/cc$. Target $Q_{P1}=1000$ and $Q_{S1}=10$.

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