AAPG Search and Discovery Article #90187 © CSPG/CSEG/CWLS GeoConvention 2013, Integration: Geoscience Engineering Partnership, 6-12 May 2013, Calgary, AB, Canada

Pore-Geometric Controls on Single-Phase Darcy Permeability of Porous Media

David C. Herrick Ph D, Yellowstone Petrophysics LLC, Cody, Wyoming, USA
davidcherrick@gmail.com

Summary

Consideration of the pore-geometric controls of DC (and low frequency) electrical conduction provides a basis for understanding the geometric properties of porous materials that determine single-phase Darcy permeability. Conductivity and permeability are both described by the same type of equation and have the same distribution of equipotential surfaces and streamlines. The fluid properties of surface-wetting and viscosity however alter the current density distribution of fluid flow dramatically. An equation describing permeability is identical to that describing conductivity with an additional term accounting for surface-wetting and viscous interaction.

Introduction

The single-phase wetting-fluid permeability or absolute permeability is one of the most important properties of porous media. Despite its importance, the physical meaning of permeability is often not completely understood. The ambiguity in meaning is due to the way it is defined. Permeability appears as a proportionality constant in the Darcy flow equation:

$$Q = k\,(\Delta P/\mu)\,(A/L); \qquad (1)$$

where Q is flow rate, k is permeability, ΔP is the applied pressure potential, μ is the fluid viscosity, A is cross-sectional area of the medium and L is its length. k is the proportionality constant required to equate both sides of the equation. It has units of cm^2, but for convenience k is usually given in Darcy units with one Darcy equaling 9.9×10^{-9} cm^2. The flow equation explicitly accounts for the potential field, fluid viscosity and the external dimensions of the medium of interest; but the internal properties of the medium are not explicitly stated; hence they are included in an undefined way in the proportionality constant, permeability. It is the role of proportionality constant that makes permeability ambiguous in a physical sense.

Physical interpretation of permeability

The flow equation (1) is only one of several such equations all of which describe an equilibrium flux in a porous medium in response to a potential gradient. For example, comparable equations can be written for DC electrical conductivity and diffusion. Since the flux in each case is present only in the pore space of a porous medium (no flux in the solid portion of the medium), the geometry of a potential gradient throughout the medium is identical for each. The potential gradient that results from the application of a potential field across the medium is only a response to the imposition of the field; the geometry of the gradient does not depend on the nature of the potential. The potential gradients for conduction, fluid flow and diffusion are identical for a given porous medium. The geometry of the potential gradient can be determined by solving Laplace’s equation for the pore geometry of the medium.

The flow equation for electrical conductivity can be expressed by Ohm’s law as

$$I = f\,(\Delta V/R_w)\,(A/L), \tag{2}$$

where I is the electric current, f is the formation conductivity factor (reciprocal of Archie's formation resistivity factor), ΔV is the applied electrical potential, R_w is the resistivity of the saturating brine and A/L is the cross-sectional area-length ratio as in equation (1). f and k are the comparable proportionality constants in equations (1) and (2) that reflect the geometry of the potential gradient as determined by the pore geometry, but in an unspecified way.

The formation conductivity factor can be separated into two components: a pore volumetric component (porosity, ϕ), and a pore geometric component (the geometrical factor, E_0), $f = \phi E_0$. Since the potential gradient geometry is the same for fluid flow and conduction, the pore geometric factors must also be the same, $k \propto f = \phi E_0$. In fact the only difference between k and f is the effects of viscosity and wetting of the pore surfaces in the case of fluid flow.

For a cylindrical straight tube or pipe, the effects of surface-wetting and viscous interaction of fluid molecules can be determined since they cause the flow-velocity profile to be parabolic. Integration of the flow rate across the tube results in the permeability of the tube $k = A/8\pi$, where A is the cross-sectional area of the tube. The tube is straight and has 100% porosity so that both geometric and volumetric factors are equal to one. In a general porous medium, however, neither are equal to one, and the effects of surface wetting and viscosity are also present. The permeability of a porous medium must account for all three.

In a porous medium pore throats, particularly the limiting pore throats, are the major determining factor of flow rate. If one makes the simplifying assumption that the pore throats in a porous medium are cylindrical, then permeability can be made physically explicit. Accounting for wetting of the pore throat surfaces, pore volume and pore geometry results in an equation for permeability to a single-phase wetting fluid of a porous medium:

$$k = (A/8\pi)\,E_0\,\phi. \tag{3}$$

One can make other assumptions about the cross-sectional geometry of pore throats and perform the integration to arrive at other parameters to replace the $A/8\pi$ term.

Experimental validation

If one makes permeability and formation conductivity factor measurements on a sample of a porous medium, then the average cross-sectional area of limiting constrictions in the medium can be calculated from (3). It can also be estimated from mercury-injection capillary pressure measurements. The two estimates can be compared to test the efficacy of equation (3) and the assumptions that went into it. Such a test was carried out using data from fifty-two samples of a wide variety of rock-types as given in the Shell Rock Catalog. The agreement is remarkable, especially considering the experimental differences in the measurements, substantiating the logic used in arriving at equation (3).

Conclusions

Permeability is a proportionality constant in the Darcy flow equation. As such it has no intrinsic meaning. By default, permeability must account for the effects of porosity, pore geometry and any interaction between a flowing fluid and pore surfaces.

The general physics describing the flux through a porous medium in which the flux is restricted to the pore-space is the same for electrical conduction, single phase fluid flow and diffusion. All three require the same pore-geometric and volumetric parameters. The effects of interaction between the flux and pore surfaces is negligible for conduction provided that there is no significant surface conduction. It is also likewise negligible for diffusion, provided that the pore space is significantly larger than the size of the diffusing particles and there is no adsorption or chemical interaction with pore surfaces. For the flow of a wetting fluid, the surface interaction must be accounted for. The surface interaction terms for wetting-fluid flow through media with simple geometries such as a cylindrical tube is well known and easily calculated.

The proportionality factor in the flow equation for electrical conduction (no surface conduction), the formation conductivity factor, requires only pore-geometric information. The formation factor can be separated into volumetric and geometric components, $f = \phi E_0$.

Permeability consists of the same geometric parameters as electrical conduction plus a term accounting for surface interaction. Hence $k = \phi E_0 S$, where S is the surface interaction term. For flow through a cylindrical tube or pipe, $S = A/8\pi$, where A is the cross-sectional area of the tube and $k = (A/8\pi) E_0 \phi$.