Seismic source wavelet estimation and sparse-spike deconvolution

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Summary

In this paper, we present an algorithm for seismic source wavelet estimation that is based on seismic time frequency spectral decomposition with matching pursuit technique. The main assumption of this algorithm is that the source wavelet is stationary for single wavelet estimation in a selected time window and that the source wavelet has normalized energy to avoid scale ambiguity between reflectivity and source wavelet amplitude.

Introduction

In seismic exploration, a short duration seismic source wavelet (pulse) is transmitted from the surface, reflected from boundaries between underground earth layers, and received by an array of sensors on the surface. With the assumption that this seismic pulse wavelet is not distorted during its propagation (perfect elastic media), the recorded seismic trace x can be modeled as the outcome of a convolution between the reflectivity sequence (e.g. layered earth model) and a source wavelet, i.e.

$$x = r * h + added\ noise \qquad (1)$$

where r refers to a time series of reflect coefficients and h refers to a source wavelet. Because the convolution of the source wavelet destroys many low and high frequencies in reflectivity, the sharp layered structures are smeared in our received traces. Therefore, removing the effect of the source wavelet's impact on our received traces is of great significance.

The deconvolution process involves separation of the components of convolution in the observed trace x. Satisfactory results were obtained when one component was known. However, when only the observation is known, the problem becomes much more difficult. In this case, uniqueness is no longer satisfied and strong hypotheses have to be made about the components. Fortunately, we have very strong reasonable *a priori* information about r (i.e. the sparse structure) that makes solving the problem possible. A further assumption of minimum phase of h and white noise of r leads to predictive deconvolution (Robinson and Treitel, 1980). However, the minimum phase assumption cannot always be satisfied in the real world. Seeking an alternative way for mixed phase wavelet deconvolution has become a topic of great interest in current research (e.g. Dondurur, 2010). Solving equation (1) without knowing both r and h is categorized as blind deconvolution and usually involves a non-linear inverse problem. Such kind of inverse problems are generally ill-posed due to a band limited source wavelet and the sparse structure of r, which makes the equation solver complicated (e.g. Lopes and Barry, 2001). When reflectors are well separated in seismic data, a sparse inverse with an optimal L1 norm can approach a unique solution. We propose a simple algorithm that is based on the homotopy method of sparse pursuit and wavelet decomposition techniques to estimate the source wavelet.

Method description

The method is based on time-frequency decomposition. In order to explain the method, first consider the case of only one reflector with the reflection coefficient γ located at time t_i and equation (1) becomes

$$x(t) = \gamma h(t - t_i) \tag{2}$$

Even in such a simple case, the solution to equation (2) is still ambiguous for the choice of the scale factor between γ and h. By assuming a stationary procedure, we can define that the energy embedded in the source wavelet is normalized to a unit, and then decompose $x(t)$ into a set of wavelets w, such as Ricker or Molet wavelets , i.e.

$$h(t) = \sum_j^p a_j w\left(t - t_u, f_j, \varphi_j\right) \delta(t_u - t_i) \tag{3}$$

where p represents number of central frequencies. Wavelets centered at both different frequencies and time constitutes a set of wavelet dictionary. Then

$$x(t) = \sum_j^p b_j(\varphi_j) w\left(t - t_u, f_j\right) \delta(t_u - t_i) + added\ noise \tag{4}$$

Where the b_j are complex coefficients that contains both factors of a_j and γ (e.g. Asif, 2008); t_u, f_j and ϕ_j are center time, peak frequency and phase of wavelets in the dictionary. By solving equation (4) for b_j, $x(t)$ can be reconstructed. The source wavelet can then be achieved by normalizing the reconstructed $x(t)$.

In an analogue to the procedure above, a seismic trace can be time-frequency decomposed by matching pursuit (e.g. Asif, 2008), where the data is reconstructed to corresponding to each reflection coefficient step by step. If wave propagation is stationary, we can expect to obtain the same source wavelet at each pursuit step. However, such a stationary assumption may not hold in a real case and the final source wavelet is taken as the average of all estimated wavelets. It is important to note that, even if a specific wavelet is used to generate a set of elements of a wavelet dictionary for decomposition, the complex coefficient that contains the amplitude and phase can adjust the shape differences between the source wavelet and the wavelet used for the decomposition.

Because of the presence of noise in seismic data, high noise levels will also be used for data fitting (in addition to the reflectivity series). In such a case (where confusion exists between noise and data) a unique solution for a dense reflector distribution may not exist. Therefore, in practice, a window needs to be selected where seismic events are relatively well separated.

In summary, our algorithm can be defined as:

a) generate a wavelet dictionary;

b) select a window where the reflectors inside the window are well separated;

c) use matching pursuit to find the wavelet decomposition at a sparse time location;

 d) calculate the residual between data and reconstructed data and go for the next sparse location.

As described above, our algorithm is actually a matching pursuit seismic time-frequency spectral decomposition and this decomposition effectively performs seismic data noise attenuation because it uses a wavelet dictionary and sparse reflectivity to match the seismic data.

Therefore, unlike many other algorithms that alternate between updating the sparse reflectivity and the source wavelet by solving ill-posed object equations (e.g. Kaaresen, 1997), our algorithm is robust for noise contaminated data.

Example

A synthetic data example will illustrate our algorithm. A mixed phase wavelet is shown in Figure1 (a), and is convolved with a well separated random reflectivity in Figure (b), which produces the synthetic observed data in Figure 1 (c) with 5% added white noise. Figure 2 shows the spectrums of source wavelet (a), reflectivity (b) and synthetic data (c), which relate to Figure 1, respectively. The wavelet dictionary is generated from a Ricker wavelet and its 90 degree rotated version. The estimated source wavelet, reflectivity and observed data are shown in Figure 3. Figure 4 shows the spectrums that correspond to Figure 3. It can be seen that both source wavelet and reflectivity are well estimated. Also, the estimated data are noise-free as expected.
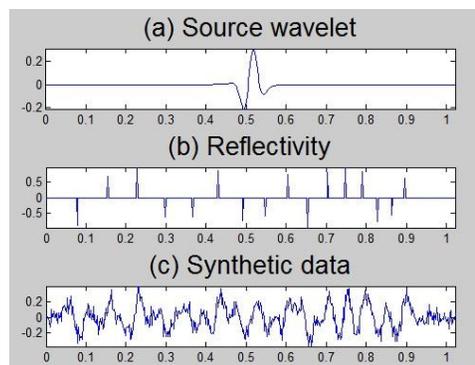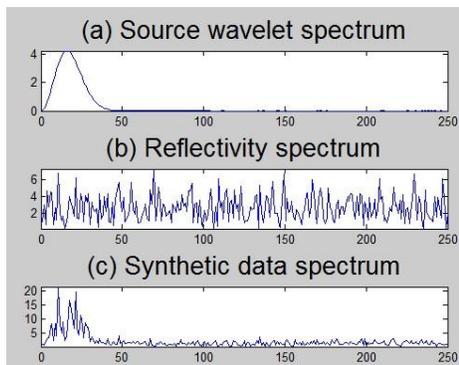
Figure 1. Input synthetic data
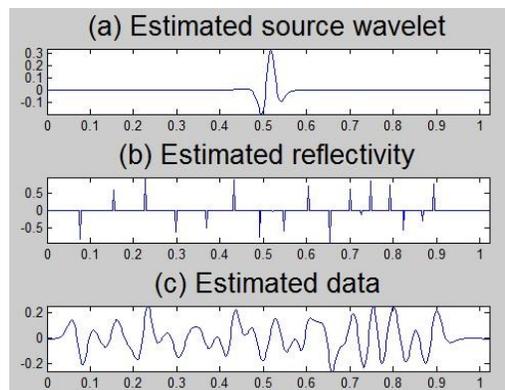
Figure 2. Spectrum of input data
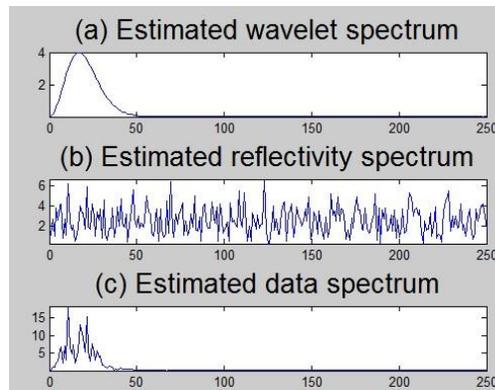
Figure 3. Estimated data

Figure 4. Spectrum of estimated data.

Conclusions

We have presented an algorithm for estimating the seismic source wavelet based on matching pursuit seismic time frequency decomposition. Because the wavelet "dictionary" is generated with different dominant frequency bands, each element in the dictionary has a different length. Together with the complex coefficient that can adjust the phase change to accommodate the source wavelet shape, we can flexibly model different source wavelet shapes. This algorithm does not involve any kind of ill-posed inverse; it works in a stable manner. Finally using the redundant property of the wavelet dictionary to fit data, this algorithm is robust for noisy data as it also works to attenuate noise.

References

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