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Robust Rank-Reduction Filters for Erratic Noise

Stewart Trickett*, Fugro Seismic Imaging, Calgary, Alberta
strickett@fugro.com
Lynn Burroughs, Fugro Seismic Imaging, Calgary, Alberta
Andrew Milton, Fugro Seismic Imaging, Calgary, Alberta

Summary

Rank-reduction filters operating on constant-frequency slices are highly effective at removing Gaussian random noise. Prestack seismic data, however, often contains spatially erratic noise which is far from Gaussian, sometimes causing these filters to give poor results. Here we describe new robust rank-reduction filters which can handle both Gaussian and erratic noise, and we present examples using real data.

Introduction

Random noise is often *erratic* – that is, it has wild values (*outliers*) that do not obey a Gaussian distribution. When we apply statistical estimators designed for Gaussian noise to such data, the results are often poor. Figure 1 shows how the arithmetic mean (the optimal estimator for errors having a Gaussian distribution) is badly skewed by a few outliers, while the median is not. Estimators like the median that are insensitive to outliers are termed *robust*.

Figure 1: The arithmetic mean is skewed by outliers, while robust estimators like the median give a reasonable answer.

Land prestack seismic traces often contain erratic noise (Claerbout and Muir, 1973). Air blast, power line noise, parity errors, isolated noise bursts (often due to deconvolved spikes, zeroes, and clips), poor quality shots, scattered shot noise, disabled or poorly coupled geophones, uncorrected polarity reversals, wind, rain, and endless other effects can lead to noise which is non-Gaussian in the spatial direction. This problem is exacerbated by two developments. First, AVO-friendly processing flows forbid the application of trace-by-trace scaling, a simple but effective means to tame high-amplitude outliers. Second, modern 3D surveys are often huge, making manual trace editing impractical.

Removing noise before stacking can improve multiple removal, AVO analysis, prestack inversion, and the final stack. It's also critical to remove severe erratic noise before prestack migration, as the migration operator will smear the noise across the section, making it impossible to correct afterwards.

Over the last ten years a rich family of random noise suppressors based on matrix rank reduction on constant-frequency slices has been developed:

- Eigenimage (Trickett, 2003)
- Cadzow, also know as SSA (Trickett, 2002, 2008; Sacchi, 2009)
- Hybrid Eigen-Cadzow (Trickett and Burroughs, 2009)

Each method works on a regular grid of traces in any number of spatial dimensions using Algorithm 1 (next page). The methods vary primarily in how they form the matrix in step 2.1. Cadzow filtering in one spatial dimension, for example, creates a Hankel matrix, and in two spatial dimensions creates a Hankel matrix of Hankel matrices. Hybrid Eigen-Cadzow concatenates Hankel matrices together. Trickett and Burroughs (2009) demonstrated these filters on prestack data.

1: *Take the Discrete Fourier Transform (DFT) of each trace.*
2: *For each frequency…*
- ***2.1***: *Insert the complex values for this frequency and all traces into a matrix.*
- ***2.2***: *Reduce the rank of the matrix.*
- ***2.3***: *Place the matrix entries back into the DFT of the traces.*

3: *Take the inverse DFT of each trace.*

Algorithm 1: Rank-reduction filtering on constant-frequency slices.

The crucial noise suppression step is 2.2, reducing the matrix rank. Typically one uses a fast approximation to a truncated Singular Value Decomposition (truncated SVD). See Trickett (2003) and Gao, Sacchi, and Chen (2011). This is optimal in that, given an *n*-by-*n* matrix **A** and a rank *k*, it generates the rank-*k* matrix **R** that minimizes

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \left| [\mathbf{A}-\mathbf{R}]_{i,j} \right|^2$$

Thus the truncated SVD is a least-squares solution, and performs well when the noise is Gaussian but poorly when it is not. Our goal is a robust rank reduction that performs well for all random noise.

Method

Many methods of robust rank reduction have been developed in the last few years. Here we combine iteratively reweighted least squares (Scales and Gersztenkorn, 1987) with weighted rank reduction (Srebo and Jaakkola, 2003), which we will call *Iteratively Reweighted Rank Reduction*, or IRRR. Given a matrix **A** corresponding to an input frequency slice **S**, Algorithm 2 shows how to calculate a robust rank-reduced matrix **R**.

The algorithm weights frequency slices rather than matrices so as not to access individual matrix elements. When combined with efficient algorithms for block-Hankel matrices (Gao, Sacchi, and Chen, 2011), we can avoid explicitly forming the matrix.

The reweighting scheme for the frequency slice **T** is critical. It must produce a solution which is *robust* (insensitive to outliers) and *efficient* (similar to the non-robust solution for Gaussian noise). We prefer *redescending* schemes such as bisquare or Hampel, as they can better handle large outliers (Maronna et al., 2006). Suppose frequency slices **S** and **T** of Algorithm 2 have samples $\{s_i\}$ and $\{t_i\}$ respectively. Then a bisquare reweighting is

$$t_i \leftarrow w\, s_i + (1-w)\, t_i$$

where

$$w = \begin{cases} [1-(u_i/\varepsilon)^2]^2 & \text{when } u_i < \varepsilon, \\ 0 & \text{otherwise.} \end{cases}$$

$$u_i = |s_i - t_i|$$

and ε is 4.7 times a robust estimate of the standard deviation of $s_i - t_i$ (Ji, 2011).

Examples

Figure 2 shows a slice through a synthetic two-dimensional grid of seismic traces having three planar events and strong erratic noise. Four different noise filters are tried. The first three – all least-squares methods – give poor results, but robust Cadzow does an excellent job of recovering the signal.

R ← Rank-reduced **A**.
Iterate until the changes in matrix **R** are small…
- **T** ← Frequency slice derived from matrix **R** using step 2.3 of Algorithm 1.
- Reweight frequency slice **T** based on **S.**
- **B** ← Matrix derived from frequency slice **T** using step 2.1 of Algorithm 1.
- **R** ← Rank-reduced **B.**

Algorithm 2: A robust rank reduction

Figure 3 shows real CDP gathers that are contaminated with high-amplitude incoherent shot noise at the very near offsets – so high of an amplitude that standard Cadzow filtering in the CMP-offset domain distorts some coherent energy. Robust Cadzow does a better job of removing shot noise and preserving coherence. The stack (Figure 4) is also improved.

Conclusions

The world is not Gaussian, and we can come to considerable grief assuming it is. Here we have described a fast and simple means to convert a rank-reduction filter into a robust filter that can deal with both Gaussian and erratic noise. The result is a novel random-noise attenuator capable of handling the diverse statistical predicaments found in prestack land seismic.

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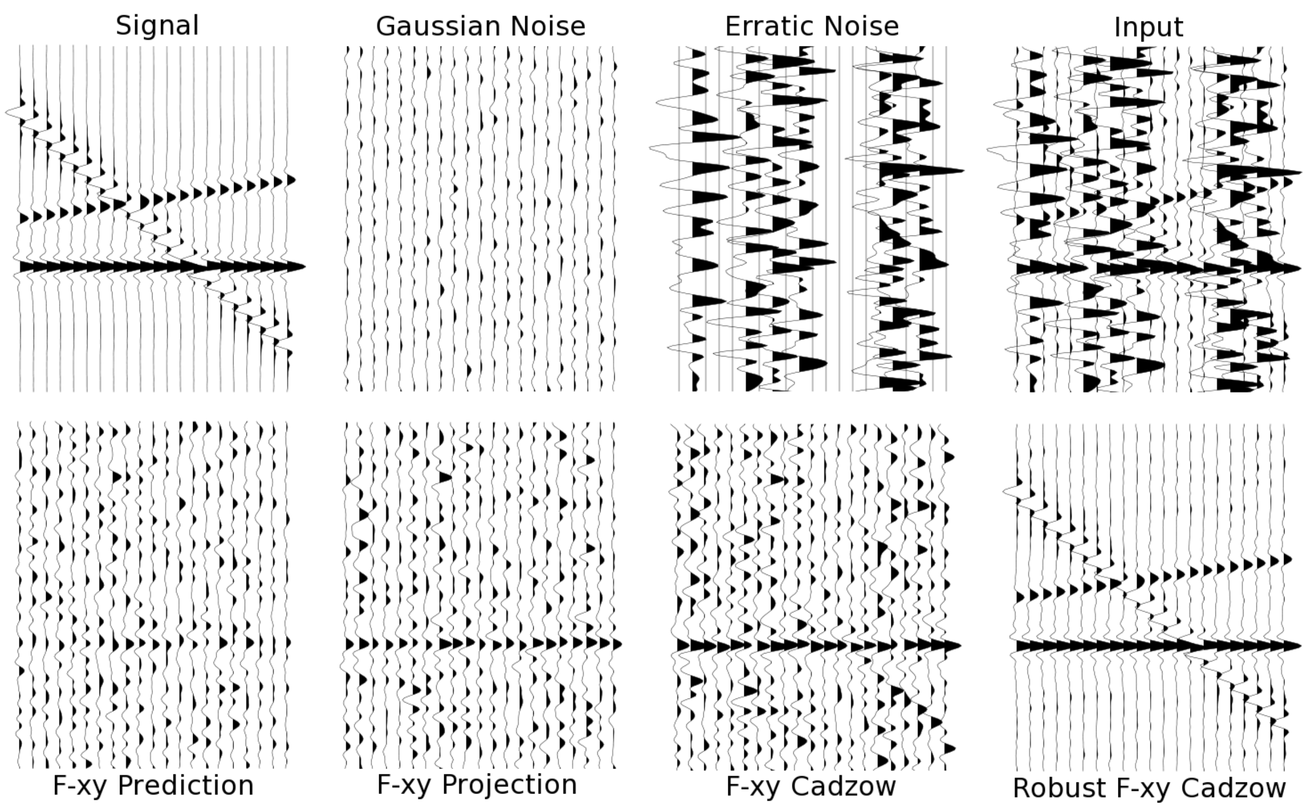

Figure 2: A synthetic model with two spatial dimensions (only a one-dimensional slice is shown). The input on the upper right is composed of signal plus mild Gaussian noise, with one-third of the traces (selected at random) contaminated with strong erratic noise. On the bottom row, four types of noise suppressors are applied to the input. Only robust Cadzow fully recovers the signal.

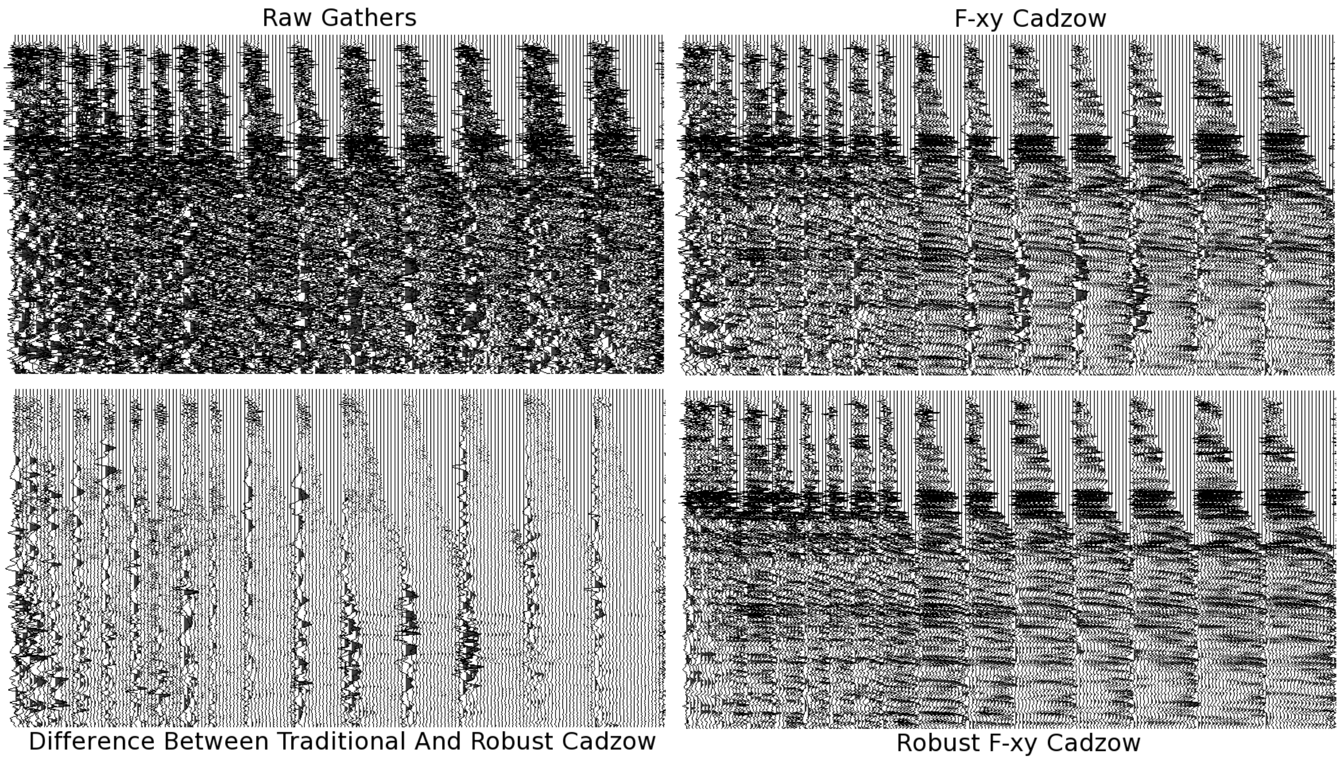

Figure 3: Top left is every fifth CMP gather from a data set with incoherent shot noise contaminating the very near offsets. Top right has conventional f-xy Cadzow applied in the CMP-offset domain. Bottom right has robust f-xy Cadzow applied instead. Bottom left is the difference between the two data sets on the right. Note how much more shot noise the robust filter has removed than the conventional Cadzow. Also note that the extreme amplitude of the shot noise may have caused the conventional Cadzow to remove a small amount of coherent energy in the centre of the gathers. Data courtesy of Connacher Oil and Gas Limited.

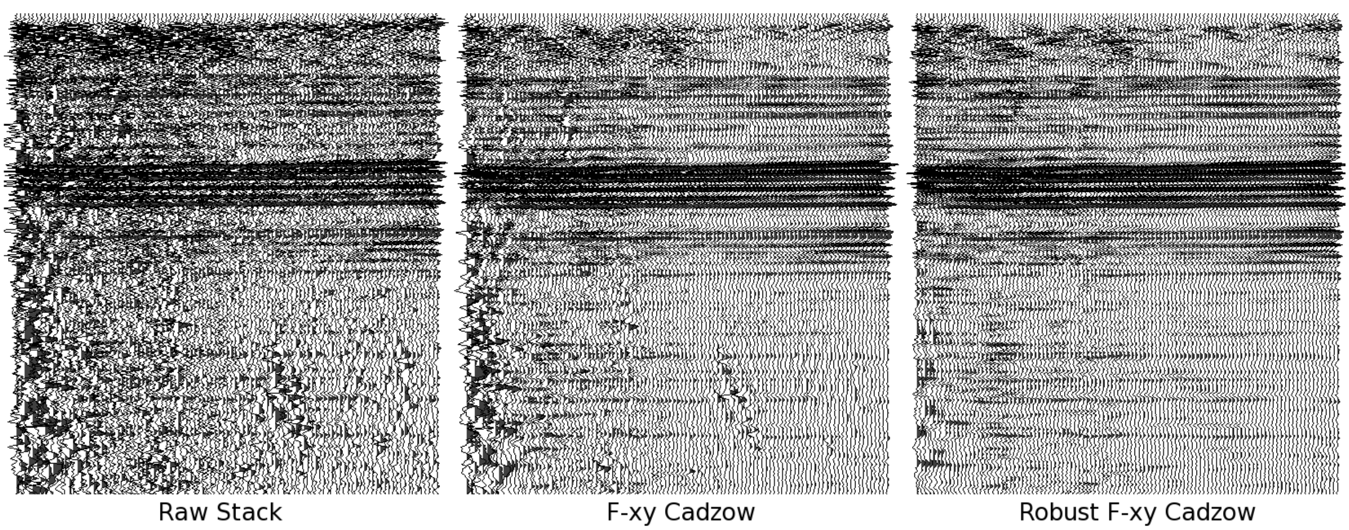

Figure 4: Stacks of the above gathers. Robust Cadzow (right) has removed more erratic noise than conventional Cadzow (centre). Data courtesy of Connacher Oil and Gas Limited.