Uncertainty Space for Microseismic Event Locations using a Double Difference Algorithm
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Summary
Double difference relocation was originally developed to improve earthquake location results. Although its application to microseismic datasets is mathematically straightforward, differences in the scale between earthquake seismology and microseismic observations and other assumptions of the original algorithm are other considerations for which we must account.

In microseismic mapping with downhole arrays, a limited number of receivers are used and the coverage of the focal sphere is often limited. For earthquake observations the size of the event cluster is much smaller than the array aperture. For many microseismic surveys, the event cluster is comparable in size, or even bigger than, the array aperture. These differences often lead to poorly-constrained and ill-conditioned systems of double difference equations. To improve the results of the relocation, we utilized two computed values, observation coverage and ray path similarity, to prioritize event-pairing in the algorithm and implemented coordinate uniform matrix normalization.

The LSQR algorithm is used for solving the system of double difference equations. Though robust and reliable, it fails to produce realistic uncertainty estimates when damping is used to constrain the double difference system. An estimation of the reliability of the new event locations is produced by introducing the observation quality of an event. Quantitative estimates of the relocation uncertainty are achieved through Monte Carlo simulations.

Introduction
The double difference relocation method is a well-established algorithm used routinely in earthquake seismology to improve relative locations of earthquake hypocenters. To achieve better locations of microseismic events, interest to the aforementioned algorithm is gaining in the industry. The double difference relocation usually creates more compact event distributions and often introduces linear features. In this paper we use different synthetic datasets to illustrate the conditions when artifacts are generated by the double difference algorithm and provide some guidelines on when the algorithm is expected to improve the overall results.

The double difference method adjusts the locations for event pairs to minimize the difference between observed and theoretical travel time differences.

\[
\frac{\partial t_k^i}{\partial m} \Delta m^i - \frac{\partial t_k^j}{\partial m} \Delta m^j = (t_k^i - t_k^j)^{\text{obs}} - (t_k^j - t_k^i)^{\text{calc}}
\]

Equation 1: Double difference. \( \frac{\partial t}{\partial m} \) - slowness vector collinear with takeoff direction measured at the source; \( \Delta m \) - adjustment vector; \( k \) – phase observed on common receiver for paired events \( i \) and \( j \).

A linear system, combined from equations for all selected event pairs, is solved using LSQR - conjugate gradient algorithm of Paige and Saunders (1982). LSQR is well suited for large and sparse double difference design matrices.
In order to minimize the impact of unknown velocity anomalies, events with similar ray paths at the common receiver are linked (Figure 1). In this case, the difference between travel times of paired events will be attributed to spatial offset between events. For this assumption to hold true, the separation between two events must be small compared to distance between the events and the receiver and the scale length of the velocity heterogeneity (Waldhauser, 2000).

Stations positioned within 200 km from the cluster centroid were used to relocate the Long Valley earthquakes (Waldhauser, 2000). The earthquake cluster had about 20 km of lateral extent. The maximum event separation threshold was set to 10 km and average distance between linked events was 0.5 km (Waldhauser, 2001). By comparison, a Barnett shale microseismic mapping example (Warpinski, 2005), had a horizontal receiver aperture of 500 meters and an event cloud extent of about 1000 meters. In case of earthquake observation size of event cloud is much smaller than observation setup aperture which is quite different for most microseismic surveys.

\[
\frac{\text{receiver aperture}}{\text{cluster size}} > 20, \text{seismology} \quad \quad \quad \quad \frac{\text{cluster size}}{\text{receiver aperture}} < 1, \text{microseismic}
\]

If the double difference algorithm parameters are downscaled from earthquake seismology to microseismic problems using average distance from event to receiver, events farther than 20 meters apart will be too far to be linked and the average expected distance between linked events would need
to be about 1 meter. These are very small values for microseismic clouds which are typically 1 km in diameter. In order to create an interconnected event network we have to use a bigger event separation threshold. On the other hand, large size of microseismic events clusters, relative to observation aperture, increases chance of linking events with very different ray paths. Taking these facts into account, we expect the typical double difference system to be less conditioned than a seismological one.

**Estimating uncertainties**

Some sources of uncertainty in the double difference relocation, potentially leading to creation of artifacts, is poor observation coverage, the linking of phases with non-similar ray paths and matrix normalization. When damping is used to regularize the naturally ill-conditioned double difference system the LSQR method greatly underestimates uncertainties. To get more reliable error estimations, a Monte Carlo simulation is applied. In our simulation, random picking errors are introduced and the event is relocated using the double difference method. About one hundred realizations are performed in order to get a representative description of the location uncertainty.

**Practical application**

The spatial distribution of the sensors has a significant impact on the location accuracy. Figure 3 shows the results of the Monte-Carlo simulation for a dual-array and triple-array setup. These results show clearly that, for the dual-array setup, linear artifacts are easily introduced in certain parts of the model. Given an ideal triple-array geometry, we do not generate these linear artifacts.

Having good observation coverage of a chosen area of interest is not necessarily enough to get a well-constrained double difference system. For real data, poor waveform quality and suboptimal array placement can lead to a reduced number of observations and incomplete coverage. To minimize the negative impact of poorly-observed events on the overall relocation quality, it is necessary to include the real observation coverage in the algorithm.

To quantify observation coverage, singular value decomposition (SVD) is performed on the takeoff vectors as seen at the event hypocenter. The take off vectors are then projected on all principal directions and maximum projections are chosen as semi axes of the observation ellipsoid. Ellipsoid size is scaled to make the major axis (A) equal to 1. After scaling, the closer minor axes B, C are to unity, the better is event observation coverage and the lesser the chance of creating linear artifacts.
Besides the observation coverage, the numerical normalization of the double difference matrix can be another source of uncertainty. The double difference matrix is normalized to improve numerical stability of LSQR. Independent column normalization, utilized in HypoDD (Waldhauser, 2000), combined with poor observations of some events leads to high relocation uncertainties, especially if the take off vectors are close to the coordinate axis. By implementing a more robust normalization, where x, y, z columns for each event are normalized using a common coefficient, the chance of creating artifacts is minimized. The new normalization, along with utilization of observation coverage and ray path similarity in the event pairing algorithm, is used to improve stability of double difference solution and minimize relocation uncertainties.

Tests on synthetic and real data have shown that the estimates of the uncertainty space, derived from the observation coverage, agrees with the results from the Monte-Carlo simulation thus providing a robust tool to assess reliability of double difference relocation. The uncertainty space, after the relative localization, often has a similar shape and direction when compared with the uncertainty of individual (non double-differenced) localization, and its size can be up to an order of magnitude smaller.

Conclusion

The double difference method generally improves location over single event localization, but before applying it to microseismic data, it is essential to ensure that all assumptions of the algorithm are met. This paper has shown some tools to quantify and minimize artifacts created by the double difference algorithm and provided a method to quantify the uncertainty after the relocation.

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References


