Harmonic Decomposition of a Vibroseis Sweep Using Gabor Analysis

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Summary

In traditional Vibroseis surveys the harmonic frequencies generated by the vibrator are seen as undesirable noise distortions. These harmonic effects cause a correlation-ghost forerunner or a tail at both positive and negative correlation times if the harmonically distorted sweep or "source signature" is used as the correlation operator. An innovative approach is proposed to decompose Vibroseis source signatures into their respective fundamental and harmonic components such that the harmonics and their higher frequencies can be used for seismic imaging or more accurate filter design. The decomposition is accomplished through the use of the Gabor transform and least squares to produce broad band estimates of the fundamental and harmonics of the Vibroseis source signature.

Composition of a Vibroseis source signature

We assume a noiseless distorted source signature $\sigma_{ds}(t)$ is the sum of the fundamental (σ_1) and all (n) harmonics

$$\sigma_{ds}(t) = \sum_{n=1}^{N} \alpha_n * \sigma_n(t) = \alpha_1 * \sigma_1(t) + \alpha_2 * \sigma_2(t) + \alpha_3 * \sigma_3(t) + ... + \alpha_n * \sigma_n(t) + ...$$

(1)

where all α_n are small time-independent convolutional (*) filters. Equation (1) provides a base for four unique methods to solve for the coefficients. These solutions are the time stationary, frequency stationary, time-dependent Gabor solution and the frequency-dependent Gabor solution. This paper will only deal with the results of the time-dependent Gabor solution.

Time-dependent Gabor solution (with stability factor!)

The Gabor transform is a nonstationary generalization of the Fourier transform (Margrave et al 2004). Applying the continuous Gabor transform to the distorted source signature as seen in equation (1) results in, $G_{ij}(\sigma_{ds}) = G_{ij}(\sum_{n=1}^{N} \alpha_n * \sigma_n)$. Here we use subscript *i* for time and *j* for frequency.

Expansion of the previous equations relies on the approximation $G_{ij}(\alpha_n * \sigma_n) \approx G_{ij}(\alpha_n)G_{ij}(\sigma_n)$ which is

justified in (Margrave et al 2011). We then define the Gabor coefficient as $G_{ij}(\alpha_n) = \hat{\alpha}_{nij}$, $G_{ij}(\sigma_{ds}) = h_{dsij}$ and $G_{ij}(\sigma_n) = h_{nij}$. For the time-dependent Gabor decomposition we solve for each time *(i)* to find the best $\hat{\alpha}_{ni}$ for *N* harmonics where

$$\begin{bmatrix} \hat{\alpha}_{1i} \\ ... \\ \hat{\alpha}_{Ni} \end{bmatrix} \approx (\beta_i^T \beta_i + \lambda b \max(\beta_i)^{-1}) \beta_i^T \begin{bmatrix} \sum_j h_{dsij} \overline{h_{1ij}} \\ ... \\ \sum_j h_{dsij} \overline{h_{Nij}} \end{bmatrix} \qquad (2)$$

and

$$\beta_i = \begin{bmatrix} \sum_j h_{1ij} \overline{h_{1ij}} & ... & \sum_j h_{Nij} \overline{h_{1ij}} \\ ... & ... & ... \\ \sum_j h_{1ij} \overline{h_{Nij}} & ... & \sum_j h_{Nij} \overline{h_{Nij}} \end{bmatrix}. \qquad (3)$$

The term β_j^T has been multiplied through equation (2) to ensure β_j is square and invertible. A stability function $\lambda b \max(\beta_j)$ is added to ensure non-zero real values are in the diagonal of the square matrix. The expression *b* is a stability factor and λ is the unit matrix. Attenuation through the earth and sample rate will limit the number of available harmonics (*N*).

Decomposing of a sweep

A base plate recorded source signature which is acting as a proxy for the "ground motion" was used to test the time-dependent Gabor decomposition. All components from the fundamental and the first harmonic (H2) to the seventh (H8) harmonic were calculated. Figure 1 shows the time domain (left) and the frequency domain (right) individual results (magenta) of time-dependent Gabor decomposition with respects to the fundamental, H2, H3, H4 and H7. The blue "sweep" in the time domain represents the successive addition of previous components producing a "vanishing" effect to the original source signature plotted in black.

Figure 2 shows the Gabor spectrum of the test source signature (upper left), the time-dependent Gabor decomposition (upper right) and error (bottom) between original and the decomposed result. The remaining signal on the error plot (bottom) appears to be ambient noise 60 db's down. While harmonics to H10 (ninth harmonic) are observable in the Gabor spectrum of the test source signature (upper left of Figure 2), present algorithm design limits successful decomposition to H8.
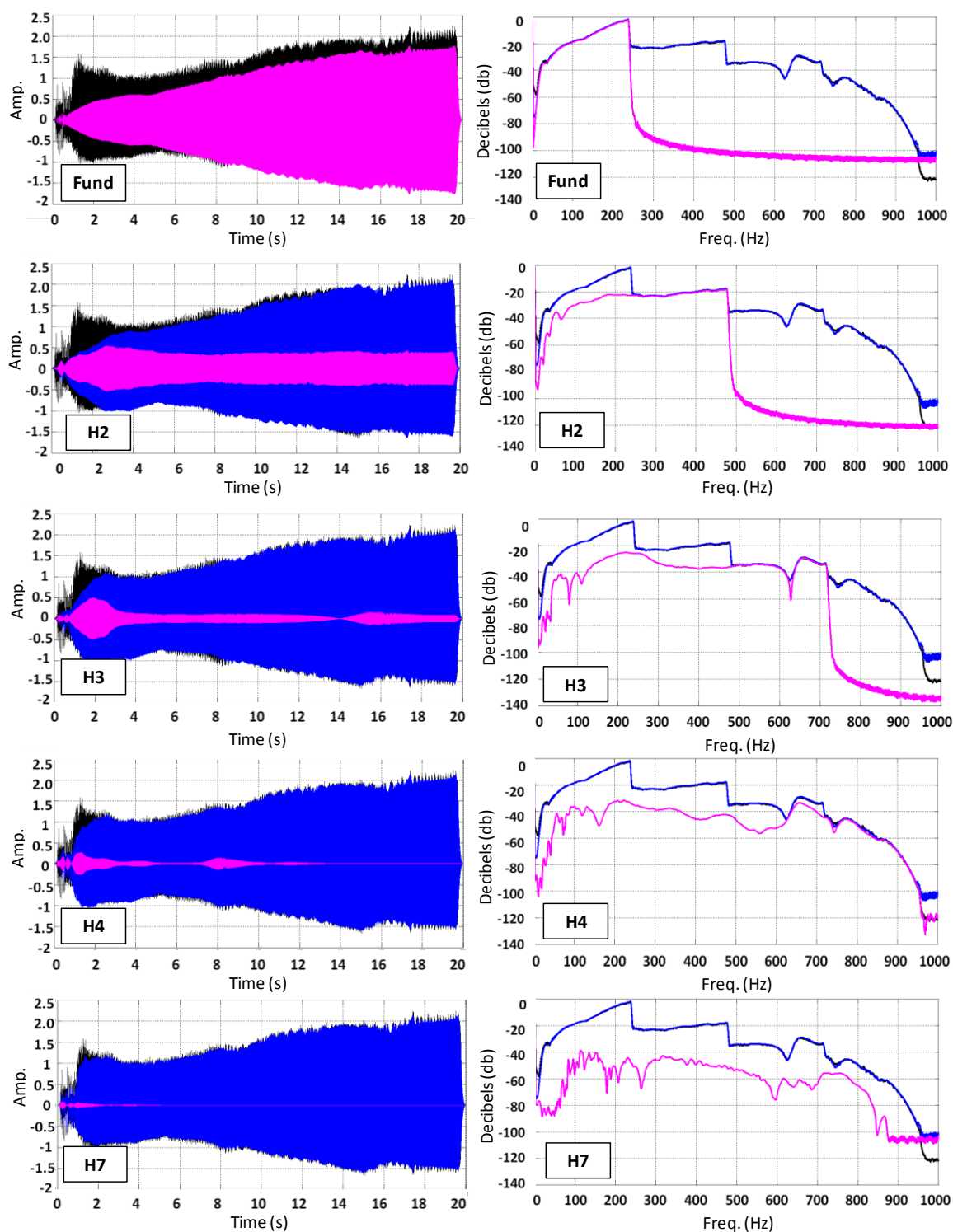
Figure 1: The individual time domain (left) and frequency domain (right) results for the time-dependent Gabor decomposition of our test source signature. The original source signature is plotted in black, successive component addition are in blue on time domain, and individual results are in magenta.
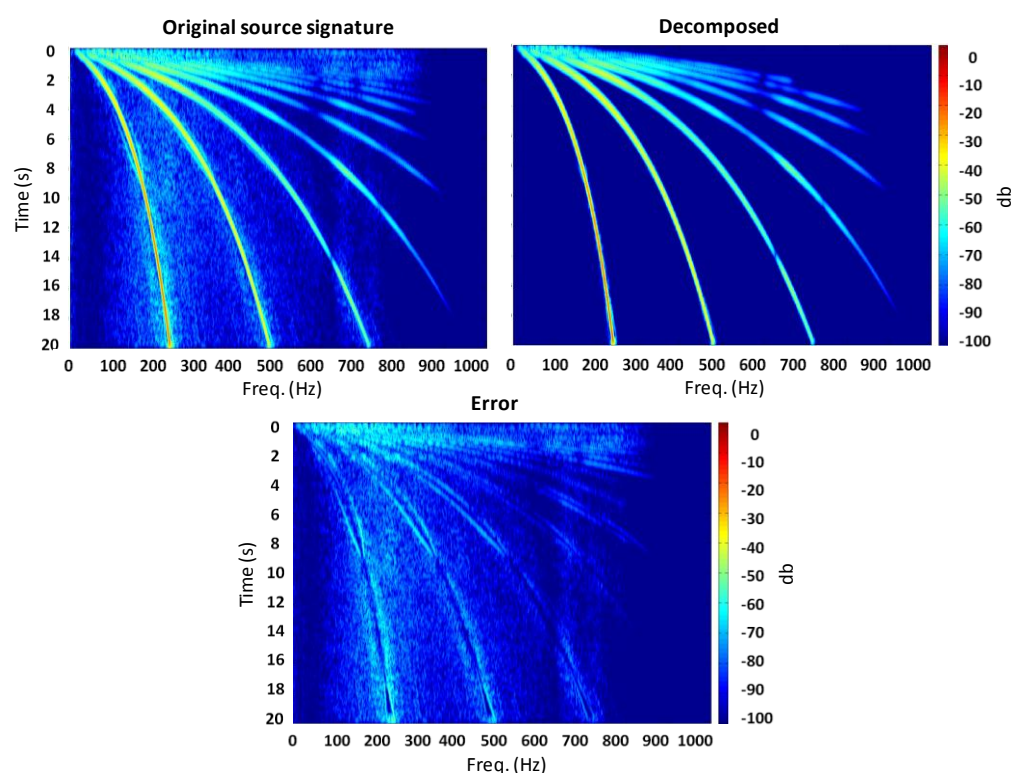
Figure 2: The Gabor spectra for our test source signature (upper left), the final decomposition result (upper right) and the error between the two (bottom).

Conclusion

Using Gabor analysis, and least squares, broad band estimates of the fundamental and harmonics within a source signature recorded at the base plate were achieved. As with all seismic imaging, decomposition is ultimately reliant on data quality of the original sweep.

Future Work

The successful decomposition of Vibroseis source signature provides a unique opportunity to attempt bandwidth expansion above the frequency range of the pilot sweep.

Acknowledgements

References

Margrave, G. F., Gibson, P.C., Grossman, J.P., Henley, D.C., and Lamoureux, M.P., 2004. Gabor deconvolution: theory and practice: EAGE. Paris France.
Margrave, G. F, Lamoureux, M. P, and Henley, D. C. Gabor deconvolution: Estimating reflectivity by nonstationary deconvolution of seismic data: Geophysics, Vol. **76**, No. 3, W15-W30.
Sheriff, A. J., and Kim W.H., 1970. The effect of harmonic distortion in the use of vibratory surface sources: Geophysics, Vol. **35**, No. 2, 234-246.