Variable-factor S-transform for time-frequency decomposition, deconvolution, and noise attenuation

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Summary
The variable-factor S-transform is a new time-frequency decomposition method, which shows better focusing than the Gabor transform and the conventional S-transform. A nonstationary deconvolution method is developed, similar to Gabor deconvolution. The method is tested on a constant Q model and shows superior results over the traditional stationary Wiener deconvolution and an improvement over the Gabor deconvolution. An *f-t-x* noise attenuation method based on the transform is developed and a synthetic example proves its effectiveness.

Introduction
Most of today's seismic data processing and analysis are based on the assumption that the seismic signal is stationary and employ an extensive use of Fourier analysis. However, due to various attenuation mechanisms of the earth, the seismic signal is not stationary. Margrave (1998) presented the theory of the nonstationary linear filtering, which is a better approximation to the physical phenomena of the seismic wave propagation in the earth over the traditional stationary convolutional model. Margrave and Lamoureux (2001) presented a nonstationary deconvolution based on the Gabor transform (Gabor, 1946). Stockwell (1996) developed the S-transform time-frequency decomposition, which improves the resolution of the Gabor transform.

The variable-factor S-transform
One drawback of the Fourier transform is that it is not localized in time. Figure 1 is a display of a particular signal and its amplitude and phase spectra. The Fourier transform tells us correctly which frequencies exist in the signal, but does not tell us when in time these frequencies exist. For nonstationary signal, whose frequency content changes over time, we need both, the time and the frequency dependence simultaneously. Gabor (1946) proposed the short-time Fourier transform, now called the Gabor transform, as a way to compute the localized spectra of nonstationary signals

$$G(\tau, f) = \int_{-\infty}^{+\infty} h(t) w(t-\tau) e^{-i2\pi ft} dt \tag{1}$$

where $w(t)$ is the analysis window, usually Gaussian, and $G(\tau,f)$ is the complex Gabor spectrum. A fundamental property of the time-frequency decomposition is the uncertainty principal, which states that the maximum possible time-frequency resolution is governed by $\Delta t \Delta \omega \geq 1/2$. Choosing a narrow window leads to a good time resolution but poor frequency resolution, while wide window leads to good frequency resolution but poor frequency resolution. Figure 3 is a display of the Gabor time-frequency decomposition of the signal from Figure 1 using windows with variable width. Figure 3 a) is the Gabor spectrum with 0.01 seconds width, which obviously is a poor choice and leads to smearing along the frequency axis. Figure 3 b) is the generated with 0.05 seconds window width, which shows a better resolution, except for the 10 and 20 Hz components. By increasing the window width to 0.1 seconds (Figure 3 c)) we achieve a good frequency resolution, however we have lost the time resolution for the 150 Hz component. By increasing the window to 0.2 seconds width we loose completely the time resolution of the high frequency components. Obviously to achieve good time-frequency we need variable width windows. Stockwell et al. (1996) proposed the S-transform, which Gaussian windowing function standard deviation is the inverse of the frequency

$$S(\tau, f) = \int_{-\infty}^{+\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 f^2}{2}} e^{-i2\pi ft} dt \qquad (2)$$

Manshinha et al (1997) introduced a constant factor k in the standard deviation of the analysis window $\sigma(f) = k / f$. By increasing the factor k they have achieved better frequency resolution with a corresponding loss of resolution in time. Figure 4 a), b), and c) is the S-transform amplitude spectra of the signal from Figure 1 with factors 1, 2, and 3. By examining the plots one can conclude that factor of 1 is a good choice to achieve both time and frequency resolution for the low frequency components, factor of 2 for the mid-range components, and factor of 3 for the high frequency components. However in the Stockwell S-transform the factor is a constant number. We propose a variable-factor S-transform, in which the factor k is a function of the frequency as well

$$\sigma(f) = \frac{k(f)}{|f|} \qquad (3)$$

$$S(\tau, f) = \int_{-\infty}^{+\infty} h(t) \frac{|f|}{\sqrt{2\pi} k(f)} e^{-\frac{(\tau-t)^2 f^2}{2k^2(f)}} e^{-i2\pi ft} dt \qquad (4)$$

Figure 4 d) is the amplitude spectrum of the same signal using linear factor from 1 at zero frequency to 6 at Nyquist. We can see good time-frequency resolution for all frequency components.

Variable-factor S-transform deconvolution

Wiener deconvolution is one of the main processes applied in today's seismic data processing. It is based on a number of assumptions (Yilmaz, 2001), a major one of which is the stationarity of the seismic trace. However, due to the various attenuation mechanisms, the seismic trace is not stationary. Based on the theory of the nonstationary filtering (Margrave, 1998), Margrave and Lamoureux (2001) developed the Gabor nonstationary deconvolution. An important step in their development is the estimation of the time-frequency spectrum of the seismic trace, which is based on the Gabor transform. We have shown that the variable-factor S-transform has better simultaneous time-frequency resolution over the Gabor transform and we have developed a nonstationary deconvolution based on the transform. We follow the same steps as the Gabor deconvolution, with the modification of the time-frequency decomposition. Figure 5 is a display of a random reflectivity sequence and the corresponding synthetic trace with a minimum-phase wavelet and Q=100. Figure 6 is the variable-factor S-transform deconvolution result. Our analysis has shown that it is superior over the stationary deconvolution plus AGC, and an improvement over the Gabor deconvolution.

F-T-X noise attenuation

Noise attenuation methods, like the f-x deconvolution, are based on Fourier analysis. Since the seismic trace is not stationary and some noise components are nonstationary and dispersive in nature as well, we propose a new *f-t-x* noise attenuation technique on NMO -corrected CMP gathers. The method involves

- transform the CMP gather from *t-x* domain to *f-t'-x* domain using the variable-factor S-transform
- for each *f*, *t'*
- define an array d(*f*=const, *t'*=const, *x*)
- for a data point d(x_i) design a complex prediction filter in *x*-direction from the surrounding samples
- keep the error between the actual and the predicted value for each d(x_i)
- substitute the actual value with the predicted one for the sample with the largest error
- next *f*, *t'*
- inverse S-transform

A synthetic CMP model is generated to test the concept (Figure 7). Figure 8 is the result of two passes of the *f-t-x* noise attenuation. The noise has been removed successfully and the primary amplitudes have not been affected.

Conclusions

The variable-factor S-transform show a better simultaneous time-frequency resolution over the Gabor transform and the traditional S-transform. The deconvolution method based on the transform is superior over the stationary Wiener deconvolution and shows an improvement over the Gabor deconvolution. The *f-t-x* noise attenuation shows a good potential and further testing is needed.

Acknowledgements

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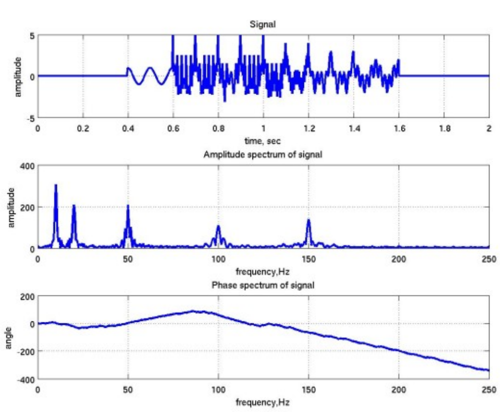

Figure 1: Signal and its amplitude and phase

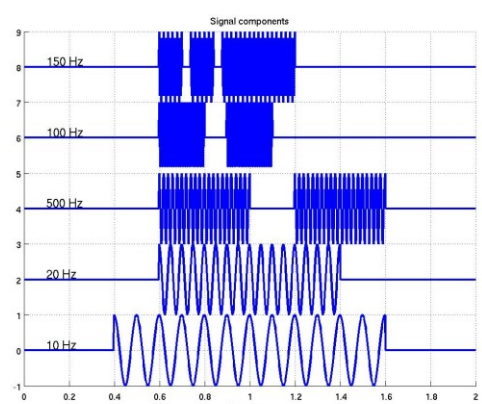

Figure 2: Sinusoids used to construct the

spectra.

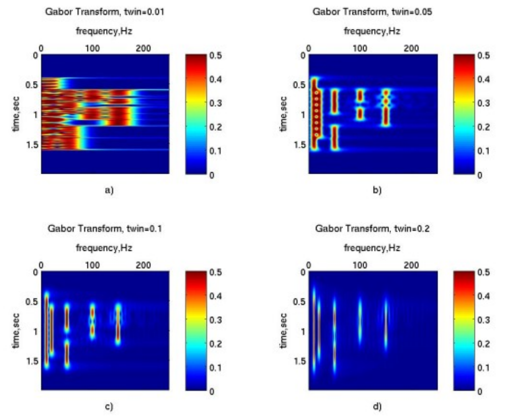

Figure 3: Gabor transform of the signal with different windows.

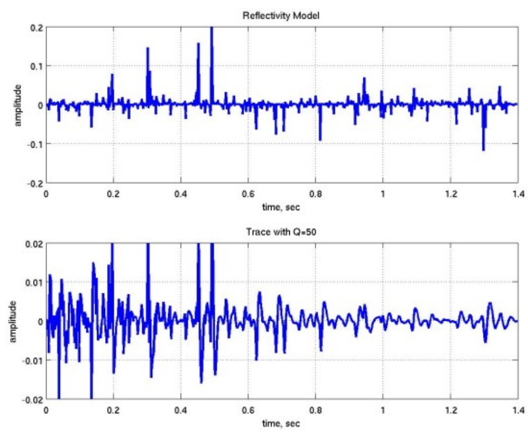

Figure 5: A random reflectivity model and a synthetic trace with Q=100.

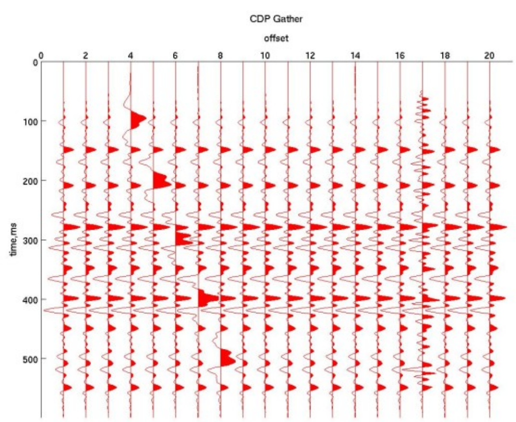

Figure 7: Synthetic CMP gather with noise.

signal.

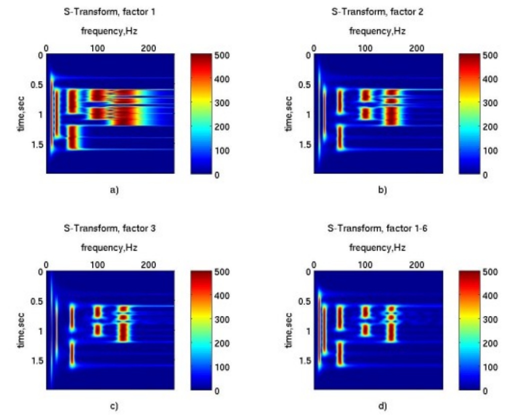

Figure 4: S-transform of the signal with different factors.

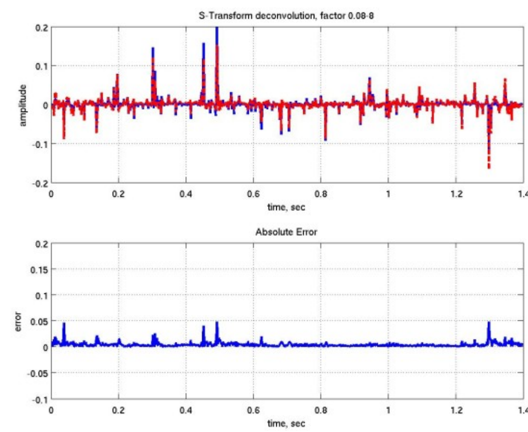

Figure 6: Variable-factor S-transform deconvolution result (in red) and the error.

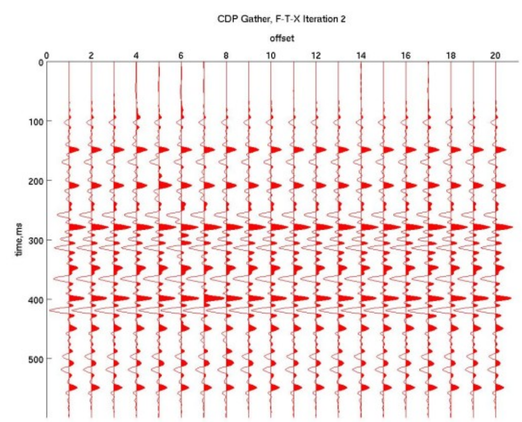

Figure 8: Synthetic CMP gather with *t-f-x* noise attenuation.