Nonstationary Predictive Deconvolution

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Summary

A partition of unity (POU) is a discrete set of usually overlapping windows that sum exactly to one for a finite interval on the real line. Multiplying a signal by the POU decomposes it into a set of temporally localized signals or Gabor slices. Applying any stationary operator to these slices and allowing the operator to depend upon the slice defines a nonstationary operator. We apply a stationary prediction operator to each slice and by summing construct a time-domain nonstationary deconvolution method based on gapped prediction filtering. We call this new method *slicedecon* because it operates directly on the individual Gabor slices. We also prescribe the construction of nonstationary autocorrelation functions as an analysis tool. We then compare *slicedecon* with the more established Gabor deconvolution or *gabordecon*. When the prediction filtering is unit-lag, we show that *slicedecon* achieves results comparable to *gabordecon* on a nonstationary (Q attenuation) synthetic. For lags greater than unity *slicedecon* appears to suppress, though not eliminate, periodicities in the nonstationary autocorrelation of a signal. Testing on a synthetic with multiples has not yet indicated any dramatic elimination of the unwanted multiple reflections.

Introduction

Predictive deconvolution (Peacock and Treitel, 1969) has long been used, with limited success, as a method of multiple suppression. The original theory uses stationary prediction filters to estimate the predictable part of a time series which is then subtracted from the original signal to give the prediction error. When the prediction distance (or lag) is one sample, the prediction error is an estimate of the reflectivity. For greater prediction lags, predictive deconvolution has been shown to remove multiples under ideal circumstances. However in general the multiple content of a seismogram is nonstationary. Two major effects are at play here. First, even simple multiples, like those from a hard water bottom, are easily shown to be periodically spaced in time only on a zero offset trace, and only if the water bottom is flat. Second, a given interface generates multiples that arrive later in time than the primary. This means that the multiple train trailing behind the primary seismic pulse grows as the pulse progresses. Taner (1980) proposed predictive deconvolution in the tau-p domain as a remedy for the first effect. Later other similar ideas have been tested such as predictive deconvolution in the radial trace domain (Perez and Henley, 2000).

Recently, we have developed a nonstationary spiking deconvolution in the Gabor domain (e.g. Margrave and Lamoureux, 2001, Margrave et al, 2004) which has proven very successful in dealing with the nonstationary effects of anelastic attenuation. Gabor deconvolution is a natural extension of Wiener spiking deconvolution (Robinson and Treitel, 1967) to the Gabor time-frequency domain. Here we report on our initial attempts to do something similar with the closely related predictive deconvolution. This development has been hindered due to the lack of a useful theory of gapped predictive deconvolution in the frequency domain. Here we sidestep this issue by developing the theory in the nonstationary “time-time” domain. This is an intermediate domain realized in the process of a Gabor transform after localizing a signal with a temporal window and before Fourier transformation. In our implementation, we use a partition of unity (POU) to decompose a signal into "Gabor slices" which are localized time signals that sum to recreate the original signal. Then on each Gabor slice we implement a conventional stationary prediction operator. This is not as desirable as having a true nonstationary prediction filter but it is a first approximation to one, and our approach as the correct stationary limit.

Theory

Consider a subset of the real line $s = [a,b], b > a$ and a finite set of functions, $\Omega_k(x), k \in K = [1,N]$ with the property

$$\sum_{k=1}^{N} \Omega_k(x) = 1, \forall x \in s \; . \quad (1)$$

The set $\{\Omega_k\}$ is called a partition of unity or POU. We also require that each Ω_k be nonnegative everywhere. Given such a POU, we define the *analysis window*, $g_k(x)$, and the synthesis window, $g_k(x)$, through

$$g_k(x) = \Omega_k^{p}(x), \quad p \in [0,1] \quad (2)$$

and

$$g_k(x) = \Omega_k^{1-p}(x), \quad p \in [0,1]. \quad (3)$$

Since $g_k g_k = \Omega_k$, the analysis and synthesis windows are a useful factorization of a POU that allows flexibility in the implementation of a nonstationary operator.

Using the POU concepts of the previous section, we can decompose any signal into a suite of *Gabor slices* defined by

$$s_k(t) = g_k(t) s(t), \quad (4)$$

and the signal can be reconstructed from its slices by

$$s(t) = \sum_{k=1}^{N} g_k(t) s_k(t). \quad (5)$$

The decomposition of a signal into Gabor slices provides a very flexible mechanism for nonstationary, or time-variant, signal analysis and processing. Let $T_k : L^2 \to L^2$ be any linear operator that maintains the finite energy of a signal, and the subscript *k* indicates that the operator can depend on window position. Then we define the nonstationary operator

$$(\mathbf{T}s)(t) = \sum_{k=1}^{N} g_k T_k s_k = \sum_{k=1}^{N} g_k T_k g_k s \quad (6)$$

as a natural extension of the corresponding stationary operator. For the problem at hand, let d_k^m be a predictive deconvolution operator having lag *m* and also being designed from Gabor slice *k*. Then we define nonstationary predictive deconvolution as

$$s_d = \sum_{k=1}^{N} g_k d_k^{m_k} \square s_k = \sum_{k=1}^{N} g_k d_k^{m_k} \square (g_k s) . \quad (7)$$

We have constructed a nonstationary predictive deconvolution code in Matlab based in equation (7). We call our method *slicedecon* and will use that name in the reminder of this paper. We will compare *slicedecon* to Gabor deconvolution and will use the term *gabordecon* for the latter. Features implemented in *slicedecon* include asymmetric POU's and the ability to prescribe the prediction filter parameters (operator length, lag, and stability constant) arbitrarily with time. In any particular Gabor slice, the prediction filter is stationary as described by Wiener's theory; however, the deconvolved signal is constructed by the superposition of many such slices and is therefore nonstationary. At this time, it is not obvious how quickly the deconvolution parameters can vary in the final result nor precisely how the POU controls this. This is a subject of future investigation.

Examples

As a first example of *slicedecon*, consider the results in Figure 1. This compares *slicedecon* with *gabordecon* and also compares the stationary equivalents of both algorithms *prdecon* and *fdecon*. The predictive deconvolutions were all run with unit-lag prediction distance. The input to this experiment was a synthetic seismogram with no multiples but a nonstationary constant-Q attenuation of Q=50. This is the kind of test that *gabordecon* does very well on, much better than stationary deconvolution. All of the nonstationary results are superior to any of the stationary ones. Five *slicedecon* results, for different prediction operator lengths) are shown and all are quite similar to one-another suggesting that operator length makes little difference. The stationary predictive deconvolution showed similar insensitivity to operator length and achieved less resolution than *slicedecon*. It can be proven formally that stationary spiking deconvolution (here *fdecon*) is identical to unit-lag predictive deconvolution (here the five traces above *fdecon*) and this experiment bears this out. However, the nonstationary algorithms show similar but not equivalent results. The *gabordecon* result is generally better resolved and this is presently attributed to the operator design process. In *gabordecon*, the operators at different times are designed simultaneously and all depend on one another in a physically plausible way. In *slicedecon*, we have not yet implemented a simultaneous operator design, instead each operator in each Gabor window is designed independently.

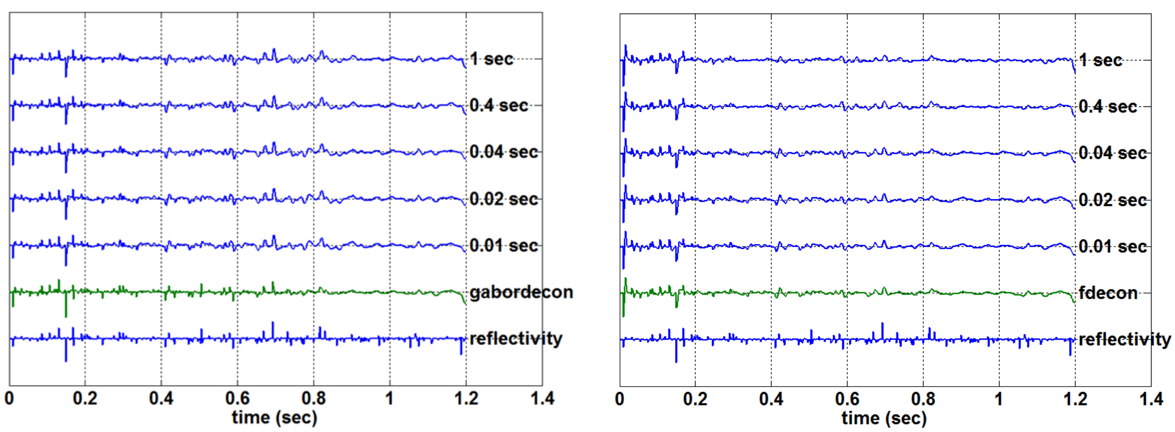

FIG. 5. (Left) A comparison between *gabordecon* and unit-lag *slicedecon*. The upper five traces are all *slicedecon* results where the numeric label on the right gives the prediction operator length. (Right) A similar comparison except that stationary algorithms are used. The input was a primaries-only synthetic seismogram with a Q=50 forward Q operator applied.

As a second example, we apply the methods to the synthetic gather shown in Figure 2a. Figures 2b and 2c show gabordecon and slicedecon results where the latter was run with unit prediction distance. Again we see that the two methods give comparable results although we note that slicedecon does not deconvolve the very early data. This is because the causal prediction operators do not have and preceding data to design on. In Figure 3a, slicedecon has bee run with a 100ms prediction gap and has produced a noticeably less whitened result than Figure 2c. Then, in Figure 3b, slicedecon was cascaded, first with a 100ms prediction distance and second with unit prediction distance. This is a common strategy with stationary predictive deconvolution and the result seems better than either previous slicedecon results especially at long offsets between 1 and 2 seconds. Finally, as a comparison, we show the result of a similar cascade of stationary predictive deconvolution and the result is clearly inferior showing badly unbalanced amplitudes and uneven whitening. Further testing (not shown) with different design windows has failed to improve the stationary result significantly.

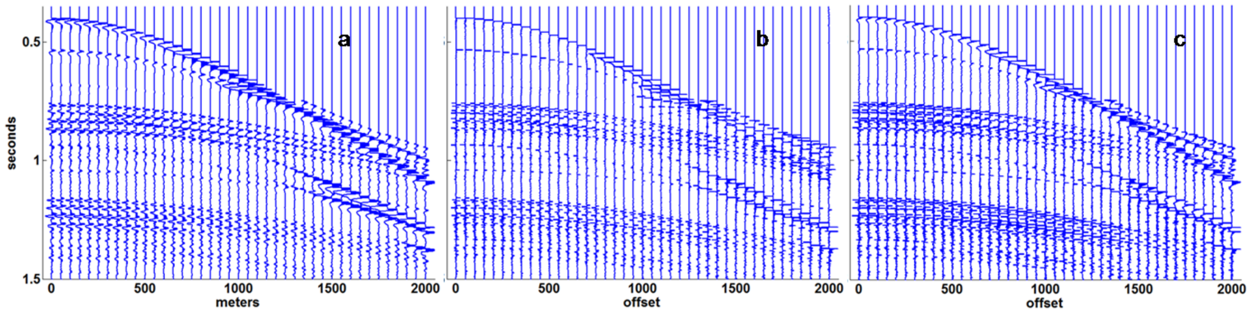

Fig. 2. (a) Synthetic offset gather containing primaries, multiples (up to 3 bounces) and mode conversion. (b) The result of *gabordecon* on the gather of (a), (c) the result of *slicedecon* in spiking mode on the gather of (a).

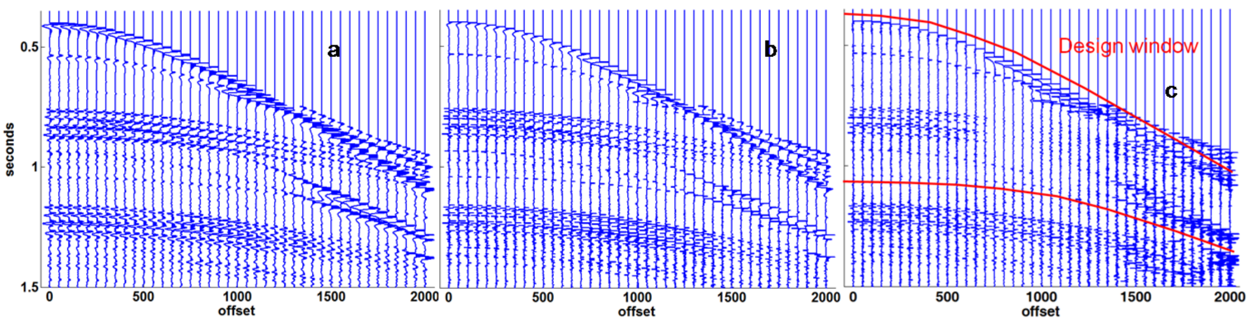

Fig. 3. (a) The result of *slicedecon* on the gather of Figure 2a using a 100ms prediction gap. (b) The result of *slicedecon* in spiking mode cascaded on top of the spiking result of Figure 3a. (c) Similar to (b) except that stationary predictive deconvolution was used. The red lines indicate the design window for the prediction operator.

Conclusions

Nonstationary predictive deconvolution compares reasonably well to *gabordecon* when the prediction distance is unity. That it is not quite as good as *gabordecon* is attributed to the fact that the deconvolution operators are designed independently rather than simultaneously. Encouraging results were obtained when cascading the algorithm with different prediction lags.

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