Seismic Modelling in 3D for Migration Testing

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Summary

A 3D modelling technique, called Rayleigh-Sommerfeld modelling, is described as an alternative to Kirchhoff modelling. Rayleigh-Sommerfeld modelling, when applied using a forward Born approximation, is shown to be the familiar phase-shift migration running in reverse. Compared to the Kirchhoff method, Rayleigh-Sommerfeld is much faster, especially on large datasets, but produces a similar response. Rayleigh-Sommerfeld is used to create an exhaustive 3D synthetic dataset which will be used for 3D migration testing. Such an exhaustive dataset, defined as having no spatial aliasing in either source or receiver gathers, can be extremely large and the efficiency of Rayleigh-Sommerfeld modelling is required to create one. The model created is the response of three horizontal reflectors embedded in a $v(z)$ medium. Consisting of 1681 source gathers, each having 1681 receivers, it is shown to be very high frequency and to contain both specular reflections and diffractions. Example 3D shot record migrations demonstrate the fidelity of the model and the high resolution of prestack migration.

Introduction

Seismic modelling plays a major role in seismic exploration and often finite-difference algorithms are chosen for their realism. However, the computational effort required for a high-fidelity result can be problematic, especially in 3D. A raytracing alternative is more efficient but does not usually include diffractions. Kirchhoff modelling is the extension of raytrace modelling to include diffractions; however, this comes at a much higher cost as each reflecting surface must be integrated across for each source-receiver pair.

Here we report on the creation of an exhaustive 3D synthetic seismic dataset by a technique which we will call Rayleigh-Sommerfeld modelling. By “exhaustive” we mean a dataset sampled sufficiently in both source and receiver positions that neither common source gathers nor common receiver gathers have any significant spatial aliasing. This is important because our primary purpose is to study the 3D acquisition footprint and we postulate that much of that footprint arises from spatially aliased data. Our paper begins with a detailed discussion of the concept of the exhaustive dataset, with special attention to the relationship between maximum frequency, velocity, and the spatial sample sizes needed to avoid aliasing. Subsequently we develop Rayleigh-Sommerfeld modelling and show that it is simply phase-shift migration in reverse. We discuss the links between Rayleigh-Sommerfeld modelling, Kirchhoff modelling, and the Born approximation. Finally, we conclude with examples of our modelled seismic data and discuss the computational cost compared to other methods. We show that this method is an effective 3D modelling technique capable of producing very high frequency responses but with no multiples, surface waves, interface waves or similar phenomena.

Theory

We assume horizontal reflecting surfaces, with arbitrarily variable reflection coefficients, embedded in a 3D medium characterized by a $v(z)$ background velocity function. Thus we explicitly decouple the reflectivity from the velocity of wave propagation, essentially invoking the first-order Born assumption that underlies modern migration methods.

The Kirchhoff method of seismic modelling is well described in the literature (e.g. Shearer, 1999) and, for a constant-velocity overburden and a given source-receiver pair involves an integration over the reflecting surface as described by

$$\psi_K\left(\underline{x}_r, \underline{x}_s\right) = \frac{ik}{4\pi}\int_S \frac{e^{ik(r_1+r_2)}}{r_1 r_2}\left[\cos(\theta_1) + \cos(\theta_2)\right]\rho\left(\underline{s}\right)d\underline{s} \tag{1}$$

where ψ_K is the Kirchhoff modelled response for a single frequency, r_1 is the distance from the source (at $\underline{x}_s$ - the underscore indicates a vector) to a point on the reflector, r_2 is the distance from the receiver (at $\underline{x}_r$) to a point on the reflector, S denotes the horizontal plane of the reflector, θ_1 is the angle of the ray from the source to the reflector, θ_2 is the angle from the reflector to the receiver, $k = 2\pi f / v$, with f being frequency and v being wavespeed, and $\rho\left(\underline{s}\right)$ is the reflectivity function. The method can be adapted to variable velocity overburden using raytracing. The 2D integration of equation (1) must be carried out over the entire computational grid for each receiver. Assuming a receiver at each grid point means that a shot record has a computational effort of $O\left(N^2\right)$ where N is the total number of grid points. Although Kirchhoff methods bring diffractions to ray theory, this comes at a computational cost that can be too great if a large dataset is to be modelled.

According to Ersoy (2007), Rayleigh and Sommerfeld proposed two theories of diffraction that are competitive with Kirchhoff theory. Called Rayleigh-Sommerfeld (RS) I and II, these two theories bracket Kirchhoff in the sense that their mean gives the Kirchhoff result (Ersoy, 2007). We present the RS I method here without derivation because it will be familiar to many involved in seismic imaging as just the phase-shift method (e.g. Gazdag, 1981). (RS II is the mathematical transpose of RS I and will not be discussed.) The calculation of a comparable result to equation (1), under the same assumptions, using RS I is given by

$$\psi_{RS}\left(\underline{x}_r, \underline{x}_s\right) = \mathbf{F}^{-1}\widehat{W}\mathbf{F}\rho\mathbf{F}^{-1}\widehat{W}\mathbf{F}\psi_{source} \tag{2}$$

where ψ_{RS} the RS I modelled response for a single frequency, $\mathbf{F}$ is the 2D Fourier transform over the lateral spatial coordinates, $\mathbf{F}^{-1}$ is its inverse, ψ_{source} is a mathematical model of the source, and $\widehat{W} = e^{iz\sqrt{k^2-k_x^2-k_z^2}}$ is the familiar phase-shift wavefield extrapolator. In words, equation (2) says (i) extrapolate the source down to the reflector by phase shift, (ii) multiply by the reflectivity function of the horizontal reflector, and (iii) extrapolate back up to the surface by phase shift. Since the integrations are accomplished by Fourier transform, RS I has a computational effort of $O(N \log N)$. Seismic modelling using the RS I diffraction theory is not new, extending back at least to Gazdag (1981), but it is often overlooked. In the next section, we demonstrate its utility to build a large 3D model.

Example

Our interest is in creating what we call an *exhaustive* 3D seismic dataset, by which we mean a dataset that shows no meaningful spatial aliasing in either source or receiver gathers. If the seismic source has an upper bandlimit of $f_{\max}$, then wavenumbers higher than $f_{\max} / v_{\min}$, where $v_{\min}$ is the slowest propagation velocity, will be evanescent and so exponentially attenuated. We use this property to choose source and receiver spacings to guarantee no aliasing from any of the reflectors in the model shown in Figure 1(a). The 100m (depth) reflector and the 180m reflector were both featureless (i.e. the reflection coeficient is not spatially variable); however, the third reflector shows the channel of Figure 1(b).

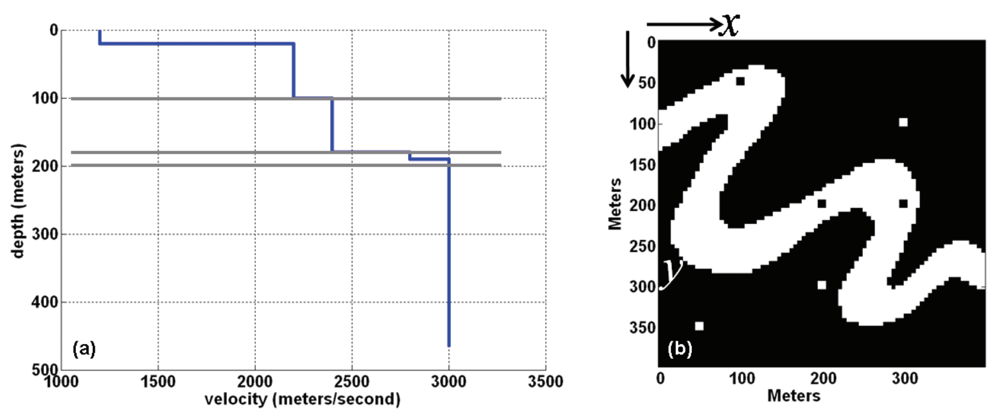

Figure1: (a) The velocity variation (blue lines) and reflectors (grey lines) for the exhaustive model. (b) The reflectivity function for the reflector at 200m depth. Black indicates a value of +0.1 and white is -0.1.

Specifically, we have chosen source and receiver spacings of 10 m and a square survey aperture of 400m x 400m. For $f_{\max}$ we used the 3db down point on the high end of the source spectrum, which is 110 Hz. Thus our dataset has 1681 receivers and the same number of sources, and 2825761 traces. Figure 2 shows a comparison between the Kirchhoff method and RS I. While Kirchhoff produces a slightly better response, the computation times strongly favour RS I.

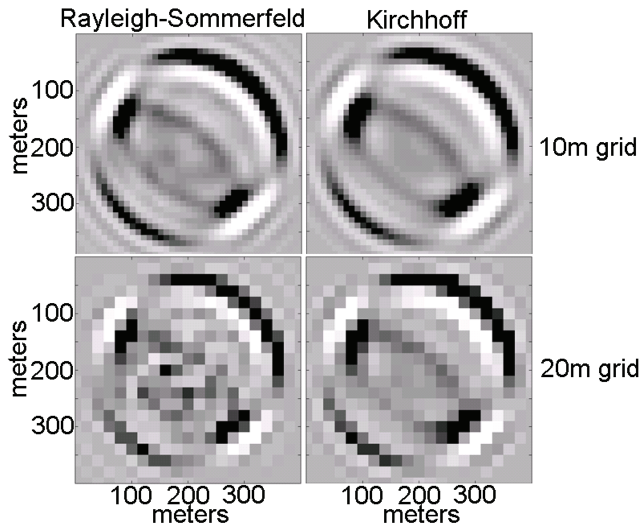

Figure 2: Comparison of RS I modelling (left) versus Kirchhoff modelling (right). Each image is a time slice through the 3D reflection response for a reflector similar to Figure 1(b). Two different grid sizes are shown and it is apparent that the Kirchhoff response is slightly better than RS I. However, the computation times for RS I are 60 s (10m grid) and 50 s (20 m) and, for Kirchhoff, 1 hour (10 m) and 5 min (20 m).

Figure 3 shows the selected 2D receiver lines from two different source locations. The RS I method has produced a very high frequency response, no discernable dispersion, and a complex diffraction/reflection response from the channel reflector.

This model was constructed to test migration algorithms. To show that the model migrates with high fidelity, two different 3D Kirchhoff shot record migrations are shown in Figure 4.

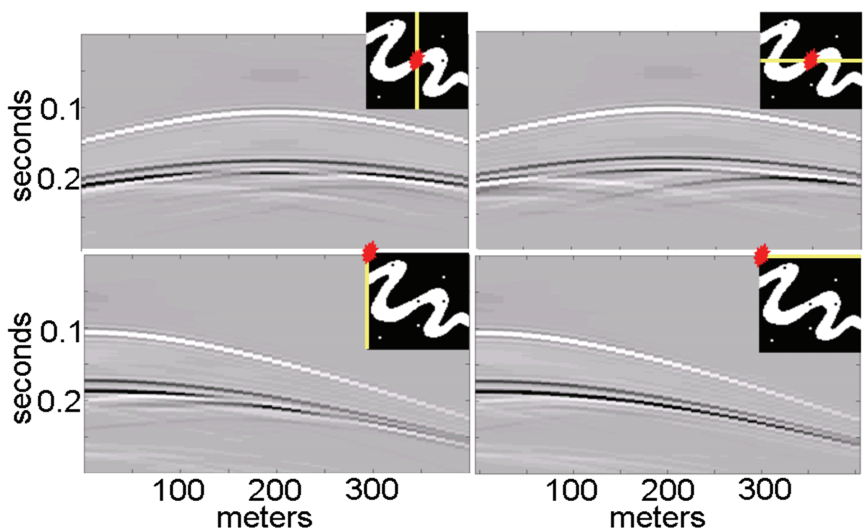

Figure 3: Four different 2D receiver lines are shown as sorted from the 3D response of the model. The small insets in the upper right of each response locates the source (red star) and the receiver line (yellow) relative to the channel reflectivity of Figure 1(b).

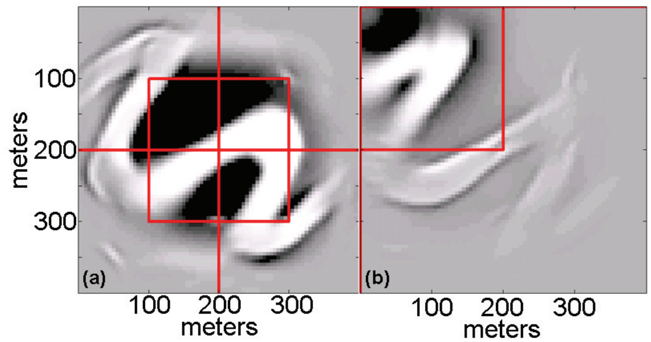

Figure 4: (a) A depth slice at the channel level of a 3D Kirchhoff migration of a source record in which the source was located in the center of the model. The red box shows the extent of the CMP coverage for this source and the red cross indicates the source location. (b) Similar to (a) except the source was at the upper left corner.

Conclusions

High fidelity 3D seismic modelling can be performed with the Rayleigh-Sommerfeld I diffraction theory. The theory is equivalent to the phase-shift method of migration, but running backwards. The results are similar to Kirchhoff theory but are computable with much less effort. Test migrations show excellent results verifying that this approach can be used to study migration and resolution.

Acknowledgements

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