Color Correction for Gabor Deconvolution and Nonstationary Phase Rotation

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Summary

Reflectivity is usually assumed to be white for conventional deconvolution algorithms. However, the reflectivity series in practice usually demonstrates a characteristicly colored spectrum, so deconvolution algorithms should be modified accordingly. This article presents a color correction method for Gabor deconvolution, proposes different ways to conduct the color correction in case of incomplete well log information, and investigates a practical way to measure and remove the residual nonstationary phase rotation remaining after deconvolution as well. The performance of proposed approaches for color correction and the practical definition of nonstationary phase rotation are evaluated using synthetic data.

Introduction

Gabor deconvolution is based on a nonstationary convolution model of the seismic trace. Margrave (1998) presented a nonstationary convolutional model, which takes the attenuation of seismic trace as a nonstationary filter applied to the stationary input. Based on this model, Margrave and Lamoureux (2002) proposed the Gabor deconvolution method, which honors the attenuation inherently and does not need additional gain correction. In practice, the reflectivity is nonwhite. Thus, the deconvolution algorithms should be modified accordingly to avoid producing distorted results. Montana and Margrave (2005) proposed a color correction method for Gabor deconvolution. This article adopts a different approach which uses the smoothed Gabor spectrum of well-log reflectivity. Since all deconvolutions are imperfect, there are generally both amplitude and phase errors in the result. Here we investigate the common practice of characterizing the error by measuring constant phase rotations. Thus, in addition to our investigation of a practical color correction method, we also investigate a practical way to measure and remove the residual nonstationary phase rotation.

Gabor deconvolution and color correction

With the assumption of minimum-phase embedded wavelet and white reflectivity, Gabor deconvolution gives estimated reflectivity in the Gabor spectral domain as Margrave and Lamoureux (2002)

$$R_G(\tau, f)_{est} = \frac{S_G(\tau, f)}{|S_G(\tau, f)| + \mu A_{max}} e^{-i\varphi(\tau, f)}, \tag{1}$$

where $S_G(\tau, f)$ is the Gabor spectrum of seismic trace, $\overline{|S_G(\tau, f)|}$ is an appropriate smoothing of $|S_G(\tau, f)|$, μ is the stability factor, and A_{max} is the maximum value of $\overline{|S_G(\tau, f)|}$. The phase $\varphi(\tau, f)$ can be calculated by Hilbert transform (over frequency)

$$\varphi(\tau, f) = H(\ln|\overline{S_G(\tau, f)}|), \tag{2}$$

where H denotes the Hilbert transform. If the assumption of white reflectivity is violated, i.e. if $|R_G(\tau, f)|$ deviates from unity significantly, then we should modify the above deconvolution algorithm. Suppose that $\hat{R}_G(\tau, f)$ is the Gabor transform of the nonwhite reflectivity $r_c(t)$ calculated from a well log, then the estimated reflectivity with color correction can be expressed as

$$\hat{R}_G(\tau, f)_{est} = \frac{S_G(\tau, f)|\overline{\hat{R}_G(\tau, f)}|}{|S_G(\tau, f)| + \mu A_{max}} e^{i\varphi_c(\tau, f)}, \tag{3}$$

where $|\overline{\hat{R}_G(\tau, f)}|$ is the smoothed result of $|R_G(\tau, f)|$, and phase $\varphi_c(\tau, f)$ is given by

$$\varphi_c(\tau, f) = H(\ln|\frac{|\overline{\hat{R}_G(\tau, f)}|}{|S_G(\tau, f)| + \mu A_{max}}|). \tag{4}$$

The key point of the color correction method is how to obtain $|\overline{\hat{R}_G(\tau, f)}|$. If sufficient well log information is available, $|\overline{\hat{R}_G(\tau, f)}|$ can be directly obtained from the Gabor spectrum of $r_c(t)$. However, in practice, the well log is usually incomplete and limited to some depth interval, which corresponds to only a part of the seismic trace. On this occasion, we need to use the limited well log to estimate a complete $|\hat{R}_G(\tau, f)|$, which should be of the same size as $S_G(\tau, f)$ in time-frequency domain. There may be different ways to achieve this. One way assumes that the color feature of nonwhite reflectivity is temporally stationary and then models $|\overline{\hat{R}_G(\tau, f)}|$ as

$$|\overline{\hat{R}_G(\tau, f)}| = a_0 + a_1 f + a_2 f^2, \tag{5}$$

in which the coefficients can be determined by a polynomial approximation of the Fourier amplitude spectrum of $r_c(t)$. As an alternative, another way infers $|\hat{R}_G(\tau, f)|$ from the Gabor spectrum of $r_c(t)$, based on an assumption that the color feature of nonwhite reflectivity is smoothly time variant. So, $|\overline{\hat{R}_G(\tau, f)}|$ can be expressed as

$$|\overline{\hat{R}_G(\tau, f)}| = a_0(\tau) + a_1(\tau)f + a_2(\tau)f^2, \tag{6}$$

For the calculation of coefficient curves $a_i(\tau)$ ($i = 1, 2, 3$), when the log is temporally shorter than the data, we first use equation (6) to approximate the Gabor amplitude spectrum of $r_c(t)$, then make a constant extrapolation in time to the corresponding coefficient curves to get $a_i(\tau)$.

Nostationary phase rotation

After applying the deconvolution, there will be an imperfect match between the estimated reflectivity and the true reflectivity, which we characterize by a nonstationary phase rotation. The successful removal of nonstationary phase rotation is related to how to define and measure it properly. We define the nonstationary phase rotation of a signal $s(t)$ as

$$\hat{s}_\theta(t) = s(t)\cos\theta(t) + s_{\pi/2}(t)\sin\theta(t), \tag{7}$$

where $\theta(t)$ is the time-variant phase rotation, and $s_{\pi/2}(t)$ is the 90 degree phase rotated version of $s(t)$. Our measurement of nonstationary phase rotation is conducted by least-squares analysis in Gabor windows.

Then, the nonstationary phase rotation can be removed either in Gabor windows or directly based on equation (7).

Examples

A 0.85s long reflectivity series, calculated from a well log, was used to test the color correction method. Figure 1 shows the reflectivity series and its' amplitude spectrum. The Gabor amplitude spectrum of the reflectivity series is shown in Figure 2. The real reflectivity is nonwhite and its color feature is time-variant.

A synthetic attenuated seismic trace was created by applying a forward Q filter to the nonwhite reflectivity, and then convolving the result with a source wavelet. For the examples in this article, the Q value is 50, and the source wavelet is a minimum phase wavelet with a dominant frequency of 40Hz. Using the complete well-log reflectivity shown in figure 1 and a frequency band of 10-150Hz, a colored deconvolution was conducted (equation 3), and the result is shown in Figure 3. Conventional Gabor deconvolution gave an obviously enlarged estimation from 0.5s to 0.8s compared to the true reflectivity series, which corresponds to the low magnitude area of the Gabor spectrum shown in Figure 2. With color correction, the estimated result is very close to the true reflectivity. Figure 4 shows the results of color correction with a frequency band of 10-60Hz, which indicates that color correction may lose its advantage when frequency band is quite limited. In case of incomplete well log, the results of practical ways to conduct color correction are illustrated by figure 5 and 6. In figure 5, the estimated reflectivity was improved assuming a stationary nonwhite reflectivity. Further improvement of estimation is apparent in figure 6 when the time-variant feature of nonwhite reflectivity is honored.

The nonstationary phase rotation between the estimated reflectivity and true reflectivity is shown in figure 7, which is measured in Gabor windows using the reflectivity series shown in figure 2. The result of color correction has a smaller phase rotation compared with that of conventional Gabor deconvolution. Then, phase correction is applied to the estimated reflectivity in Gabor windows (method 1) and based on equation (7) (method 2) respectively. And then the remaining phase rotation with respect to true reflectivity is re-measured and shown in figure 8. For the estimated results of color correction, the effect of phase correction is much better than those of conventional Gabor deconvolution, which may indicate that phase correction can achieve better results when the amplitude spectra of signals are close to each other. In addition, for phase correction, method 2 obtains a slightly better result compared with method 1, which means that the definition of nonstationary phase rotation as equation (7) is, to some degree, practical.

Conclusions

In practice, the reflectivity is usually nonwhite, and the color features can be time-variant. In presence of nonwhite reflectivity, conventional Gabor deconvolution gives a distorted estimation, which corresponds to the low amplitude area of the Gabor spectrum of nonwhite reflectivity. Color correction can significantly improve the reflectivity estimation for Gabor deconvolution, whose effect is subject to the available frequency band and completeness of well log information. To address the incomplete well log information, different approaches are proposed to conduct the color correction, which are of practical use. Testing on synthetic data shows that all these approaches can improve the reflectivity estimation to some degree. A practical way to define nonstationary phase rotation is investigated, and further work needs to be conducted on this topic.

Acknowledgements

References

Margarve G. F., 1998, Theory of nonstationary linear filtering in the Fourier domain with application to time-variant filtering: Geophysics, 63, 244-259

Margrave, G. F. and Lamoureux, M. P., 2002, Gabor deconvolution: CREWES Annual Research Report, 13.
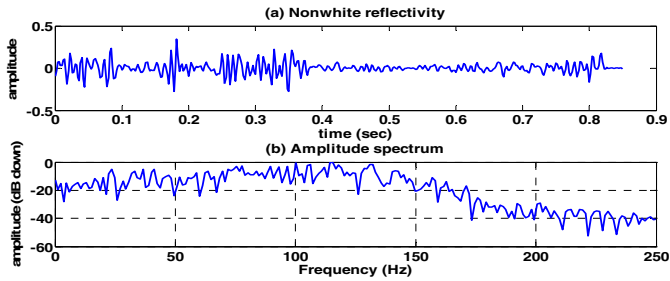
Figure 1. (a) Nonwhite reflectivity calculated from well log. (b) Fourier amplitude spectrum of nonwhite reflectivity.
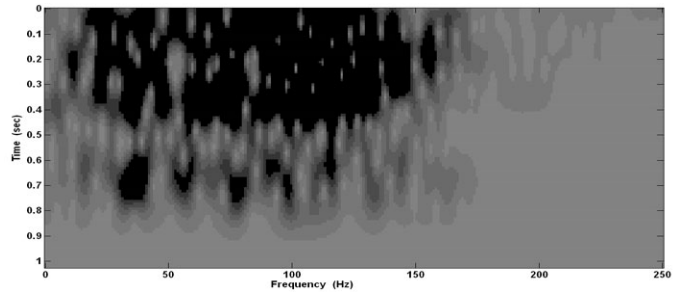

Figure 2. Gabor amplitude spectrum of nonwhite reflectivity shown in figure 1(a).
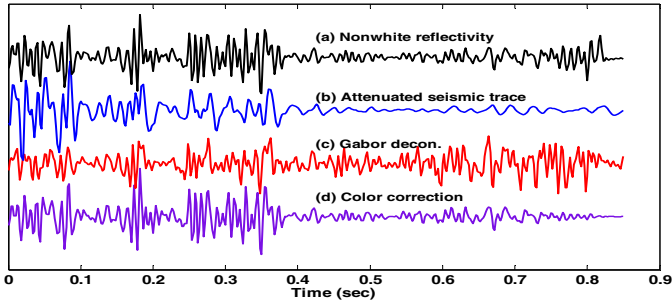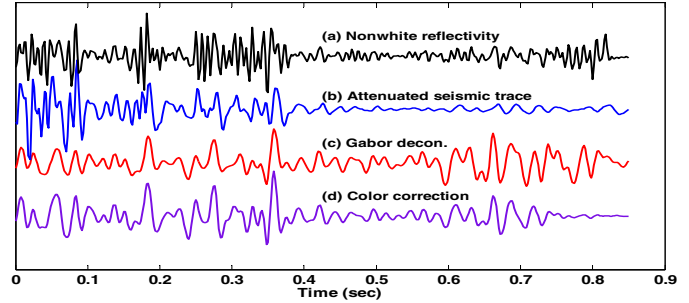

Figure 3. Gabor deconvolution with a frequency band of 10-150 Hz. (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Estimated reflectivity without color correction. (d) Estimated reflectivity with color correction using the complete well log shown in figure 1.

Figure 4. Gabor deconvolution with a frequency band of 10-60 Hz. (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Estimated reflectivity without color correction. (d) Estimated reflectivity with color correction using the complete well log shown in figure 1.
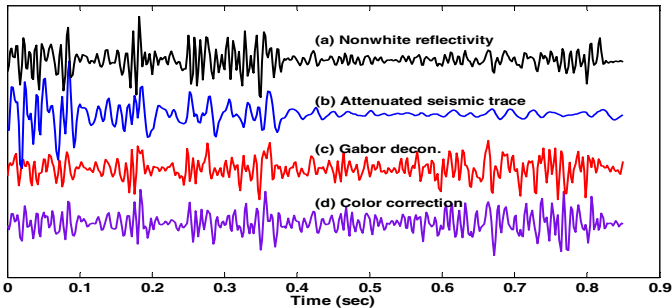

Figure 5. Color correction using the 0.2s-0.6s part of the well log shown in figure 1 (1). (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Estimated reflectivity without color correction. (d) Estimated reflectivity with color correction using equation (3) and (5).
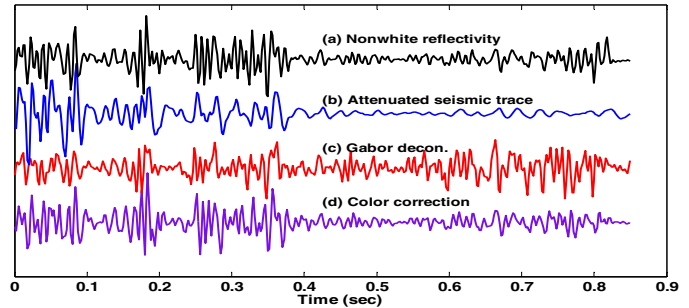

Figure 6. Color correction using the 0.2s-0.6s part of the well log shown in figure 1 (2). (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Estimated reflectivity without color correction. (d) Estimated reflectivity with color correction using equation (3) and (6).
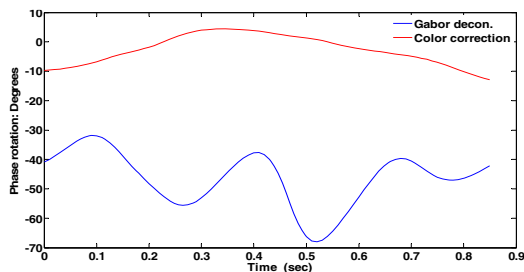

Figure 7. Measurement of nonstationary phase rotation between estimated reflectivity and true reflectivity using the reflectivity series shown in figure 3.
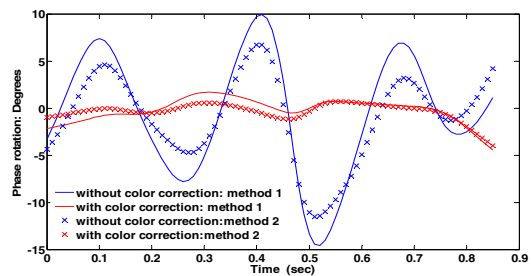

Figure 8. Remaining nonstationary phase rotation between estimated reflectivity and true reflectivity shown in figure 3 after phase correction.